Calculus 1 Unit 1 worksheet: The epsilon delta definition of a limit

Name _____ Date _____ Period _____

1. Let $f(x) = -4x - 7$, so that $\lim_{x \to 5} f(x) = -27$.

(a) Find the interval of inputs x whose outputs satisfy $|f(x) + 27| < 0.12$.

(b) Find the largest δ that guarantees an output within 0.12 of -27.

(c) Suppose $\delta = 0.06$ is used instead. Find the smallest ε for which this δ works.

2. In each statement, $\lim_{x \to a} f(x) = L$. Decide whether the statement is true for every such f, a and L. Give a reason, or a counterexample with numbers.

(a) If $\delta = 0.2$ works for $\varepsilon = 0.1$, then $\delta = 0.15$ works for $\varepsilon = 0.1$.

(b) If $\delta = 0.2$ works for $\varepsilon = 0.1$, then $\delta = 0.2$ works for $\varepsilon = 0.05$.

3. Let $f(x) = \dfrac{2x^2 - 3x - 2}{x - 2}$ for $x \neq 2$.

(a) Prove from the definition that $\lim_{x \to 2} f(x) = 5$.

(b) Which step of your proof uses the condition $0 < |x - 2|$, and not only $|x - 2| < \delta$?

4. Zoe uses $\delta = 7$ for the limit $\lim_{x \to 8} \sqrt[3]{x} = 2$. This item runs the definition backward: the window is fixed, and the question is which tolerances it meets.

(a) Find the smallest ε for which her δ works.

(b) Zoe notices that to the right of 8 every output is within 0.5 of 2, and says her δ also works for $\varepsilon = 0.5$. Find an allowed input that shows she is wrong.

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5. Prove that $\lim_{x \to 4} \dfrac{x}{x-2} = 2$. Then find the δ your rule gives for $\varepsilon = 0.3$.

6. Let $f(x) = \dfrac{x^2 - 4}{|x - 2|}$ for $x \neq 2$.

(a) Write $f(x)$ without absolute value bars for $x > 2$ and for $x < 2$.

(b) Use the definition to show that no number L is $\lim_{x \to 2} f(x)$.

7. Grace writes this scratch work for $\lim_{x \to -2}(3x^2 + x) = 10$: "$|3x^2 + x - 10| = |x + 2|\,|3x - 5|$. Agree that $\delta \leq 1$, so $-3 < x < -1$. At $x = -1$ the second factor is $|3(-1) - 5| = 8$, so $|3x - 5| < 8$, and $\delta = \min\left(1, \dfrac{\varepsilon}{8}\right)$ works."

(a) For $\varepsilon = 0.08$ her rule gives $\delta = 0.01$. Find an allowed input whose output is not within 0.08 of 10.

(b) Find her mistake, and write a correct proof that $\lim_{x \to -2}(3x^2 + x) = 10$.

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