Calculus 1 Unit 1 worksheet: Comparing growth rates at infinity

Name _____ Date _____ Period _____

1. Find each limit with the ladder, writing ∞ for a quotient that grows without bound. Name the rung comparison you use.

(a) $\lim_{x\to\infty} \dfrac{x^{3/2}}{(\ln x)^6}$

(b) $\lim_{x\to\infty} \dfrac{\ln(x^3)}{\sqrt[4]{x}}$

(c) $\lim_{x\to\infty} \dfrac{x^8\, 2^x}{3^x}$

(d) $\lim_{x\to\infty} \dfrac{\ln(x^5)}{\ln(x^2)}$

2. Find the limit. Decide first which term on the top and which on the bottom grows fastest.

$$\lim_{x\to\infty} \frac{6x^4 \ln x + x^4}{3x^4 \ln x + 4x^4}$$

3. Rank these six functions from slowest growing to fastest. Justify each neighboring pair in your ranking with the limit of a quotient.

$$x^{2.1}, \quad \frac{2^x}{x^{10}}, \quad 100x^2, \quad (\ln x)^{50}, \quad \frac{x^3}{\ln x}, \quad x^2 \ln x$$

4. Find each limit. Use a substitution that turns it into a quotient from the ladder.

(a) $\lim_{x\to\infty} \dfrac{\ln(\ln x)}{\ln x}$

(b) $\lim_{x\to\infty} x^6 e^{-x^2}$

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5. Layla tracks two videos she posted online. Her made-up models give the total views t days after posting as $A(t) = 900t^2$ for the first video and $B(t) = 5 \cdot 1.1^t$ for the second.

(a) Find $\lim_{t \to \infty} \dfrac{B(t)}{A(t)}$. Which model grows faster?

(b) Find $A(30)$ and $B(30)$, to the nearest whole view. Explain why this does not contradict part (a).

(c) If both models held forever, what fraction of the two videos' combined views would belong to the first video in the long run? That is, find $\lim_{t \to \infty} \dfrac{A(t)}{A(t) + B(t)}$.

6. Samir makes three claims. For each, say whether it is true or false. Support a true claim with a limit of a quotient, and a false one with a counterexample or the correct limit.

(a) If f and g grow at the same rate, then $\lim_{x \to \infty} \big(f(x) - g(x)\big) = 0$.

(b) $3^{x/2}$ grows slower than 2^x.

(c) $\ln(x^{100})$ grows faster than $\ln x$.

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