Calculus 1 Unit 1 worksheet: Limits at infinity and horizontal asymptotes

Name _____ Date _____ Period _____

1. Find each limit. Divide the top and the bottom by the highest power of x in the bottom, then use the basic limit and the limit laws. If a limit is not a number, write ∞ or $-\infty$ to say how the outputs grow without bound.

(a) $\displaystyle\lim_{x\to\infty} \frac{8x^2+3x}{5-2x^2}$

(b) $\displaystyle\lim_{x\to-\infty} \frac{1-4x^3}{x^2+6}$

(c) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x}+3x^2}{6x^2-x\sqrt{x}}$

(d) $\displaystyle\lim_{x\to\infty} \frac{(3x+1)^2(x-3)}{2x^3+1}$

2. Let $f(x)=\dfrac{2x+7}{x-1}$. This exercise proves $\lim_{x\to-\infty} f(x)=2$ from the definition.

(a) Show that $|f(x)-2|=\dfrac{9}{1-x}$ for every $x<1$.

(b) For the tolerance $t=0.001$, is $N=-8{,}000$ far enough out, so that $|f(x)-2|<0.001$ for every $x<N$? If not, find an N that works. Then find an N, in terms of t, that works for every $t>0$.

(c) On the far left, does the graph of f approach the line $y=2$ from above or from below? Explain.

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3. Find the horizontal asymptote at each end of the graph of

$$f(x) = \frac{3x + \sqrt{36x^2 + 5}}{2x + 2}.$$

Say which end each one belongs to.

4. Find each limit.

(a) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 9} - x \right)$ (b) $\lim\limits_{x \to \infty} x \left(\sqrt{x^2 + 9} - x \right)$

5. Let $f(x) = x\sqrt{\dfrac{x+5}{x-1}}$, which is defined for $x \leq -5$ and for $x > 1$.

(a) Use the two limits $m = \lim \dfrac{f(x)}{x}$ and $b = \lim \left[f(x) - mx \right]$ to find the oblique asymptote of the graph of f as $x \to \infty$.

(b) Find the oblique asymptote as $x \to -\infty$.

6. Write a formula for a function f, defined for every real x, with $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to -\infty} f(x) = -3$. Use the limit laws to show that your function has both limits.

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