Calculus 1 Unit 1 worksheet: The squeeze theorem and the limit of sine x over x

Name _____ Date _____ Period _____

1. Use the squeeze theorem to find each limit. Name the two walls you use.

(a) $\lim_{x\to 2}(x-2)^2\cos\dfrac{1}{x-2}$

(b) $\lim_{x\to 0}\dfrac{x^2}{3+\sin\frac{1}{x}}$

(c) $\lim_{x\to 0}\left(5+x\cos\dfrac{4}{x}\right)$

2. Find each limit. Rewrite each one with factors of the form $\dfrac{\sin u}{u}$ or $\dfrac{\tan u}{u}$, and say what u is.

(a) $\lim_{x\to 0}\dfrac{\sin 3x\ \sin 5x}{x\tan 2x}$

(b) $\lim_{x\to 0}\dfrac{1-\cos 4x}{\sin^2 3x}$

(c) $\lim_{x\to 0^+}\dfrac{\tan x}{\sqrt{x}}$

3. A function f satisfies $|f(x)-7|\le 3(x-2)^2$ for every x.

(a) Find $\lim_{x\to 2} f(x)$.

(b) Find $\lim_{x\to 2}\dfrac{f(x)-7}{x-2}$.

4. Find $\lim_{x\to 0}\dfrac{5x+\sin 8x}{\tan 5x}$.

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5. Find $\lim_{x \to 3} \dfrac{\sin 2\pi x}{x^2 - 9}$. (Hint: let $u = x - 3$.)

6. Priya draws a regular polygon with n sides inside a circle of radius 6 cm, with every corner on the circle.

(a) Show that the perimeter of the polygon is $P(n) = 12n \sin \dfrac{\pi}{n}$ cm.

(b) Find $P(4)$ and $P(6)$ exactly.

(c) Find $\lim_{n \to \infty} P(n)$. What length is the perimeter approaching?

7. All that Mateo knows about a function f is that $0 \le f(x) \le x$ for $0 < x < 1$. He concludes that $\lim_{x \to 0} f(x) = 0$.

(a) What can Mateo correctly conclude from his bounds, and why is his conclusion too strong?

(b) Give a function f that satisfies his bounds but has no limit at 0, and show that it has none.

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