Calculus 1 Unit 1 worksheet: The limit laws

Name _____ Date _____ Period _____

1. Find each limit. Name the law or result that lets you find it (for example the product law, the power law, or direct substitution for a rational function whose bottom is not 0).

(a) $\lim_{x \to -2} (x^3 + 3x^2 - x + 4)$

(b) $\lim_{x \to 3} (4 - x^2)^3$

(c) $\lim_{x \to 4} \dfrac{x^2 - 4}{x + 1}$

(d) $\lim_{x \to -3} \left(x + \sqrt{x^2 + 16}\right)$

(e) $\lim_{x \to \pi} x \cos x$

(f) $\lim_{x \to -1} \dfrac{3^{x^2 + x + 2}}{x + 4}$

2. Suppose $\lim_{x \to 3} f(x) = 4$ and $\lim_{x \to 3} g(x) = -5$. Find the limit below, naming each law you use, and show that the quotient law is allowed.

$$\lim_{x \to 3} \frac{4f(x) - x\, g(x)}{\big(f(x)\big)^2 + g(x)}$$

3. Find $\lim_{x \to 5} \sqrt{\dfrac{2x^2 - 1}{x - 3}}$ exactly. Say which law or result each step uses, and why each one is allowed.

4. Suppose $\lim_{x \to -3} \big[f(x) + g(x)\big] = -1$ and $\lim_{x \to -3} \big[f(x) - g(x)\big] = 9$. Nothing else is known about f and g.

(a) Write $f(x)$ using only $f(x) + g(x)$ and $f(x) - g(x)$. Use the laws to explain why $\lim_{x \to -3} f(x)$ exists, and find it.

(b) Find $\lim_{x \to -3} g(x)$ and $\lim_{x \to -3} f(x)\, g(x)$.

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5. For each limit, find the limits of the top and of the bottom, and decide whether the quotient law applies. If it does, give the limit. If it does not, say why, and say whether you can already tell that the limit does not exist. (A $\dfrac{0}{0}$ form is finished by the algebra of the next lesson; here only name it.)

(a) $\lim_{x\to 0} \dfrac{\cos x + 1}{x^2 + 4}$ (b) $\lim_{x\to 2} \dfrac{x^2 + 1}{\sqrt{x+2} - 2}$ (c) $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x^2 - 4}$

6. Yuna finds a limit this way:

$$\begin{aligned}\lim_{x\to 0} x^2 \left(\frac{3}{x^2} + 1\right) &= \lim_{x\to 0} x^2 \cdot \lim_{x\to 0} \left(\frac{3}{x^2} + 1\right) \\ &= 0 \cdot \lim_{x\to 0} \left(\frac{3}{x^2} + 1\right) = 0\end{aligned}$$

(a) Which step uses a law whose hypothesis fails? Explain.

(b) Find the limit correctly.

(c) Is the same first step allowed for $\lim_{x\to 1} x^2 \left(\dfrac{3}{x^2} + 1\right)$? Find that limit.

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