Calculus 1 Unit 1 worksheet: One sided limits and when a limit exists

Name _____ Date _____ Period _____

1. Diego rents a kayak. For a rental of t hours the shop charges $C(t) = 12 + 20t$ dollars when $0 < t \le 4$, and $C(t) = 24 + 17t$ dollars when $t > 4$. Find $\lim_{t \to 4^-} C(t)$, $\lim_{t \to 4^+} C(t)$ and $C(4)$, and decide whether $\lim_{t \to 4} C(t)$ exists. What do your answers say about rentals a few minutes shorter and a few minutes longer than 4 hours?

2. Find each one-sided limit from the rule that governs that side. Then find the limit from the other side, and decide whether the two-sided limit exists.

(a) $\lim_{x \to 0^-} \dfrac{|3x| - x}{x}$

(b) $\lim_{x \to 2^-} \left(x^2 - \lfloor x \rfloor^2\right)$

(c) $\lim_{x \to 2^+} \dfrac{x^2 - 4x + 4}{|x - 2|}$

3. Let $f(x) = \dfrac{|x^2 - 16|}{x - 4}$. Find $\lim_{x \to 4^-} f(x)$ and $\lim_{x \to 4^+} f(x)$, and decide whether $\lim_{x \to 4} f(x)$ exists. Then find $\lim_{x \to -4} f(x)$.

4. Let $f(x) = \sqrt{x^2 - 9}$.

(a) Find the domain of f.

(b) For each of $a = 3$, $a = -3$ and $a = 5$, say which limit of f at a is meaningful (left, right or two-sided) and find it.

(c) Let $g(x) = \sqrt{x^2 - 9}$ for $x \ge 3$ and $g(x) = 6 - 2x$ for $x < 3$. Find both one-sided limits of g at 3, and decide whether $\lim_{x \to 3} g(x)$ exists.

(d) Find $\lim_{x \to -3} g(x)$.

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5. Mei rides an elevator in an office tower whose floors are 4 meters apart; the lobby is floor 0. When the floor of the car is h meters above the lobby floor, the display shows the floor number

$$D(h) = \left\lfloor \frac{h}{4} \right\rfloor, \qquad 0 \le h \le 48$$

(a) Find $\lim_{h \to 20^-} D(h)$, $\lim_{h \to 20^+} D(h)$ and $D(20)$. Does $\lim_{h \to 20} D(h)$ exist?

(b) Find $\lim_{h \to 27} D(h)$.

(c) Find every height a with $0 < a < 48$ at which $\lim_{h \to a} D(h)$ does not exist.

(d) At $h = 0$ and at $h = 48$, which one-sided limit is meaningful, and what is it?

6. (a) Write a formula for a function f, defined for every real x, with all of these properties: $\lim_{x \to 1} f(x) = 2$; $f(1) = 5$; $\lim_{x \to 4^-} f(x) = 0$; and $\lim_{x \to 4} f(x)$ does not exist. Check each property from your formula.

(b) Could a function with these four properties also have $\lim_{x \to 4^+} f(x) = 0$? Explain using the criterion.

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