Calculus 1 Unit 1 worksheet: Infinite limits and vertical asymptotes

Name _____ Date _____ Period _____

1. Find each limit. Write ∞ or $-\infty$ when the outputs grow without bound, and write "does not exist" when the two sides disagree. Show the sign of the top and of each factor of the bottom.

(a) $\lim_{x \to 2^+} \dfrac{3x+1}{2-x}$

(b) $\lim_{x \to 5^-} \dfrac{x-8}{x^2-25}$

(c) $\lim_{x \to 3} \dfrac{6-x^2}{x^2-6x+9}$

(d) $\lim_{x \to 1} \dfrac{2x+3}{x^3-1}$

(e) $\lim_{x \to -2^+} \dfrac{1-x}{x^2+x-2}$

2. Let $f(x) = \dfrac{x+2}{(x+1)^2(x+4)}$. Find $\lim_{x \to -1} f(x)$, and find the two one-sided limits of f at $x = -4$. Explain the signs.

3. Find each limit, and say which result you use.

(a) $\lim_{x \to 0^-} \dfrac{2-\cos x}{\sin x}$ and $\lim_{x \to 0^+} \dfrac{2-\cos x}{\sin x}$

(b) $\lim_{x \to 2^-} \dfrac{x}{\sqrt{4-x^2}}$. Why is no limit from the right asked for in (b)? Is the line $x = 2$ a vertical asymptote of the graph?

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4. (a) Write $\dfrac{\sin 3x}{x^2}$ as $\dfrac{\sin 3x}{3x} \cdot \dfrac{3}{x}$, and use it to find $\lim\limits_{x \to 0^-} \dfrac{\sin 3x}{x^2}$ and $\lim\limits_{x \to 0^+} \dfrac{\sin 3x}{x^2}$. Name the results you use.

(b) Find $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x^3}$.

(c) Is the line $x = 0$ a vertical asymptote of either graph?

5. Let $g(x) = \dfrac{2}{\sqrt[3]{x-1}}$.

(a) Find a δ in terms of M that proves $\lim\limits_{x \to 1^+} g(x) = \infty$ from the definition.

(b) Give the δ your answer to (a) gives for $M = 100$, as a fraction.

(c) Find $\lim\limits_{x \to 1^-} g(x)$, and show that the same δ answers the bound M there.

6. All that is known about three functions near 4 is this: $\lim_{x \to 4} f(x) = \infty$, $\lim_{x \to 4} g(x) = -2$, and $\lim_{x \to 4} h(x) = 0$ with $h(x) > 0$ for every $x \neq 4$ near 4. Find each limit as $x \to 4$, or write "no conclusion" if the facts given do not decide it. Name the result you use.

(a) $f(x) + g(x)$ (b) $f(x)\,g(x)$ (c) $\dfrac{g(x)}{h(x)}$

(d) $f(x)\,h(x)$ (e) $\dfrac{g(x)}{f(x)}$ (f) $f(x) - \dfrac{1}{h(x)}$

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