Calculus 1 Unit 4 worksheet: The chain rule

Name _____ Date _____ Period _____

1. Find each derivative with the chain rule.

(a) $y = (4 - 3x)^6$

(b) $f(x) = (2x^3 + x)^{-2}$

(c) $g(x) = \sqrt[3]{x^2 + 7}$

(d) $y = e^{3x^2 - x}$

(e) $h(x) = 5^{\cos x}$

2. Let $y = u^3 - 6u$, where $u = x^2 + 4x$. Use $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$ to find $\dfrac{dy}{dx}$ when $x = -1$.

3. Let $f(x) = 2^x$ and $g(x) = x^2$.

(a) Find $(f \circ g)'(x)$ and $(g \circ f)'(x)$.

(b) Find $(f \circ g)'(1)$ and $(g \circ f)'(1)$. Does the order of composition matter?

(c) Check $(g \circ f)'(x)$ a second way: write $(g \circ f)(x)$ as a single exponential with one base and differentiate it.

4. Ben wrote these three derivatives, and each one is wrong. For each, say what Ben did wrong and give the correct derivative.

(a) $\dfrac{d}{dx}\left[(x^2 + 1)^4\right] = 4(x^2 + 1)^3$

(b) $\dfrac{d}{dx}\left[e^{\sin x}\right] = e^{\cos x}$

(c) $\dfrac{d}{dx}\left[\sqrt{x^3 + 1}\right] = \dfrac{1}{2\sqrt{3x^2}}$

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5. In Maya's chemistry class, a made-up model says that a reaction runs at $R = 5 \cdot 2^{(T-20)/10}$ micromoles per second when the solution is at T degrees Celsius. Maya warms the solution so that its temperature t minutes after she starts is $T = 20 + 4t$ degrees Celsius.

(a) Use $\dfrac{dR}{dt} = \dfrac{dR}{dT} \cdot \dfrac{dT}{dt}$ to find how fast the reaction rate is rising at $t = 5$. Give the exact value with units, then use a calculator and round to three decimal places.

(b) In a second run Maya warms the solution twice as fast, so $T = 20 + 8t$. How fast is the reaction rate rising at the moment the solution reaches 40 degrees Celsius?

6. The functions f and g are differentiable, with these values.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-3	2	5
2	1	6	4	-2
4	9	2	1	3

(a) Find the derivative of $f(x^2)$ at $x = 2$.

(b) Find the derivative of $\sqrt{f(x)}$ at $x = 4$.

(c) Let $P(x) = g(x)\, f(g(x))$. Find $P'(1)$.

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