Calculus 2 Unit 1 test: Integration techniques

Name _____ Date _____ Period _____

12 questions, 51 points

1. Evaluate $\int_0^1 x^2 \arctan x \, dx$ exactly. (4 points)

2. For whole numbers $n \geq 0$, let $I_n = \int_0^1 x^n e^{-x} \, dx$. (4 points)

(a) Use integration by parts to show that $I_n = nI_{n-1} - \frac{1}{e}$ for $n \geq 1$.

(b) Find I_0, and use the formula in (a) to find I_3 exactly.

(c) Without integrating, explain why $\frac{1}{4e} \leq I_3 \leq \frac{1}{4}$, and check your answer to (b) against it.

3. Evaluate $\int_0^{\pi/4} (2\cos x + \cos 3x)^2 \, dx$ exactly. (4 points)

4. Evaluate $\int_1^{1+2\sqrt{3}} \frac{(x-1)^2}{\sqrt{x^2-2x+5}} \, dx$. You may use $\int \sec^3\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$. (4 points)

5. The two integrals have the same denominator but need different methods. Find (a) $\int \frac{x^3+x}{x^4+2x^2+5} \, dx$ and (b) $\int \frac{x}{x^4+2x^2+5} \, dx$. (4 points)

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6. Evaluate $\displaystyle\int_0^{\pi/4} \frac{dx}{\cos^2 x + 4\sin^2 x}$. (4 points)

7. Evaluate $\int_0^{\pi/4} \tan^2 x \sec x \, dx$ exactly. Show the integration by parts that the odd power of secant needs. (5 points)

8. Yuna's reading lamp has a leading-edge dimmer. In a made-up model, over each half cycle the line voltage is $V = 170\sin\theta$ volts for $0 \le \theta \le \pi$, where θ is the phase angle. The dimmer blocks the voltage for $0 \le \theta < \alpha$ and passes it for $\alpha \le \theta \le \pi$. The lamp's power at each instant is proportional to V^2, so the share of full power that reaches the lamp is

$$\frac{\displaystyle\int_\alpha^\pi \sin^2\theta \, d\theta}{\displaystyle\int_0^\pi \sin^2\theta \, d\theta}.$$

(4 points)

(a) Find $\int_0^\pi \sin^2\theta \, d\theta$.

(b) Find the share when $\alpha = \dfrac{\pi}{3}$, exactly, and then use a calculator and round to three decimal places.

(c) Find the switch-on angle α that gives exactly half of full power.

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9. In a distribution center, Andre tests an automated pallet cart as it eases into a loading bay. In a made-up model its velocity is

$$v(t) = \frac{12}{t^2 + 5t + 6}$$

meters per second, t seconds after the test starts. (4 points)

(a) Find the exact distance the cart travels in the first 3 seconds.

(b) Find the exact time at which the cart has traveled $12 \ln \frac{6}{5}$ meters.

10. For a constant a, let

$$f(x) = \frac{ax^2 - 4x + 8}{x(x-2)^2}.$$

(4 points)

(a) Write the partial fraction form of f, and find each constant in terms of a.

(b) Find the value of a for which an antiderivative of f contains no term $\ln|x-2|$. For that a, evaluate $\displaystyle\int_3^4 f(x)\,dx$ exactly.

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11. Jenny wants $I = \int e^{2x} \sin x \, dx$. Her first step takes $u = \sin x$ and $dv = e^{2x}\, dx$:

$$I = \frac{1}{2} e^{2x} \sin x - \frac{1}{2} \int e^{2x} \cos x \, dx.$$

For the new integral she takes $u = e^{2x}$ and $dv = \cos x \, dx$:

$$\int e^{2x} \cos x \, dx = e^{2x} \sin x - 2I.$$

Substituting, she gets $I = I$. (5 points)

(a) Her algebra has no slip. Explain why her second step learned nothing.

(b) Find I correctly.

(c) Evaluate $\int_0^{\pi} e^{2x} \sin x \, dx$ exactly.

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12. Jasmine is laying out a practice field for her club. A circle of radius 10 meters is painted around a center spot, and a straight line painted 5 meters from the spot cuts off part of the circle. The grass in the part of the disk beyond the line is to be replaced. (5 points)

(a) Put the spot at the origin and the line at $x = 5$. Write the area of the part of the disk beyond the line as a definite integral, and evaluate it exactly with a trigonometric substitution.

(b) Show that a line at distance d from the center of a circle of radius r, with $0 \le d < r$, cuts off a piece of area $r^2 \arccos \dfrac{d}{r} - d\sqrt{r^2 - d^2}$.

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