Calculus 2 Unit 1 worksheet: Choosing an integration technique

Name _____ Date _____ Period _____

1. Each integrand becomes a basic form after a rewrite (cancel a factor or use an identity). Rewrite first, then integrate.

(a) $\displaystyle\int \frac{\sin 2x}{\cos x}\,dx$ (b) $\displaystyle\int \frac{\cos 2x}{\cos x - \sin x}\,dx$ (c) $\displaystyle\int \frac{e^{2x}-1}{e^{x}-1}\,dx$

2. Find $\displaystyle\int \frac{dx}{1+e^{x}}$ by two routes. (a) Multiply the top and bottom by e^{-x}, then substitute. (b) Substitute $u = e^{x}$, then use partial fractions. (c) Show that your two answers differ by a constant, and find that constant.

3. Find $\displaystyle\int \frac{dx}{x\,(5+\sqrt{x})}$ for $x > 0$.

4. Each integral needs a substitution first and then integration by parts. Find (a) $\displaystyle\int \frac{\ln(\ln x)}{x}\,dx$ for $x > 1$; (b) $\displaystyle\int \sin 2x\, e^{\sin x}\,dx$.

5. Both integrals contain $\sqrt{x^2+4}$. Find (a) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$ and (b) $\displaystyle\int \frac{x^2}{\sqrt{x^2+4}}\,dx$. (c) In one sentence, say why (a) needs no trigonometric substitution but (b) does.

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6. The function $\dfrac{e^x}{x}$ has no elementary antiderivative. Let $E(x) = \displaystyle\int_1^x \frac{e^t}{t}\,dt$. Use a calculator and Simpson's rule with four subintervals to estimate $E(3)$, rounded to three decimal places, and say why $E(3)$ has a value at all.

7. Take $g(x) = x^2$ in the Liouville criterion, so $\displaystyle\int f(x)e^{x^2}\,dx$ is elementary exactly when some rational function R has $R' + 2xR = f$. (a) For $f(x) = 2x^2 + 1$, find such an R of the form $R(x) = ax + b$, and use it to find $\displaystyle\int \left(2x^2+1\right)e^{x^2}\,dx$. (b) For $f(x) = 1$, show that no rational R has $R' + 2xR = 1$, and conclude that $\displaystyle\int e^{x^2}\,dx$ is not elementary. Look first at a zero of the denominator of R, then at a polynomial R.

8. Let $F(x) = \displaystyle\int_0^x \sin\left(t^2\right)\,dt$ for $0 \le x \le 3$; $\sin\left(x^2\right)$ has no elementary antiderivative. (a) Find the open intervals in (0, 3) on which F increases and on which it decreases, and the x-values of its local maximum and local minimum there. (b) Find the x-values of the inflection points of the graph of F in (0, 3). Give exact values.

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