Calculus 2 Unit 1 worksheet: Integration by partial fractions

Name _____ Date _____ Period _____

1. Find each integral by partial fractions. Divide first where the fraction is not proper.

(a) $\displaystyle\int \frac{2x+5}{x^2-x-2}\,dx$

(b) $\displaystyle\int \frac{7}{3x^2-5x-2}\,dx$

(c) $\displaystyle\int \frac{x^3+x^2}{x^2+2x-3}\,dx$

2. Evaluate $\displaystyle\int_0^2 \frac{2x-1}{x^2+3x+2}\,dx$. Give the exact value.

3. Each bottom has a repeated linear factor.

(a) Evaluate $\displaystyle\int_3^4 \frac{x^2-4x+13}{x^3-3x^2+4}\,dx$ exactly. (Look for a zero of the bottom among small whole numbers.)

(b) Find $\displaystyle\int \frac{x^2+1}{(x-1)^3}\,dx$.

4. The bottom of this integrand has a zero at a small whole number. Evaluate the integral exactly, and explain why the sign of your answer makes sense.

$$\int_{-1}^{0} \frac{3x^2+3x+4}{x^3+x^2-2}\,dx$$

5. The bottom of this integrand is a repeated irreducible quadratic. Find the integral.

$$\int \frac{x^3-x^2+9x-1}{\left(x^2-2x+5\right)^2}\,dx$$

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6. A rational function f has the antiderivative

$$F(x) = 2\ln|x+1| - \frac{3}{x-2} + \arctan x.$$

(a) Write f as a sum of partial fractions.

(b) Write the bottom of f, as one fraction in lowest terms, in factored form. If you were given f as one fraction, how many unknown constants would its decomposition have, and which of them could cover-up find directly?

(c) On which intervals is F an antiderivative of f?

7. Yuna runs a carton-folding machine at a packaging plant. After a restart, in a made-up model, it folds cartons at the rate

$$r(t) = \frac{12t^2 + 18t}{t^2 + 3t + 2}$$

hundred cartons per hour, t hours after the restart.

(a) Find the exact number of hundreds of cartons folded in the first 2 hours. Then use a calculator to give the number of cartons, rounded to the nearest whole carton.

(b) Use your work in (a) to explain why the rate stays below $1{,}200$ cartons per hour for all $t > 0$.

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