Calculus 2 Unit 5 worksheet: The ratio and root tests

Name _____ Date _____ Period _____

1. Use the ratio test to decide whether each series converges absolutely or diverges. Simplify $\left|\dfrac{a_{n+1}}{a_n}\right|$ before taking the limit L.

(a) $\sum_{n=1}^{\infty} \dfrac{n^2\, 3^n}{n!}$

(b) $\sum_{n=1}^{\infty} \dfrac{n!}{1 \cdot 4 \cdot 7 \cdots (3n-2)}$

(c) $\sum_{n=1}^{\infty} \dfrac{n!}{100^n}$

(d) $\sum_{n=1}^{\infty} \dfrac{(-2)^n\, n!}{n^n}$

(e) $\sum_{n=1}^{\infty} \dfrac{e^{2n}}{n^5\, 7^n}$

2. Use the root test to decide whether each series converges absolutely or diverges. Give the limit L of $\sqrt[n]{|a_n|}$. You may use $\sqrt[n]{n} \to 1$.

(a) $\sum_{n=1}^{\infty} \left(\dfrac{4n-1}{3n+5}\right)^n$

(b) $\sum_{n=1}^{\infty} \dfrac{3^n\, n^4}{n^n}$

(c) $\sum_{n=1}^{\infty} \left(\sqrt[n]{n} - \dfrac{1}{2}\right)^n$

(d) $\sum_{n=1}^{\infty} \dfrac{(-5)^n}{n^2\, 2^{n+3}}$

3. For each series, show that the ratio test gives $L = 1$. Then decide whether the series converges by another test, named, and say whether any convergence is absolute or conditional.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n\, 2^n + 1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

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4. Let S be the sum of the series below, and S_N its N th partial sum.

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$$

(a) Find the ratio $\dfrac{a_{n+1}}{a_n}$ and its limit, and state what the ratio test concludes.

(b) Show that the ratios decrease as n grows. Use the ratio tail bound with $N = 5$ to bound $S - S_5$, and give S_5 as a fraction.

(c) Show that $\dfrac{n}{(n+1)!} = \dfrac{1}{n!} - \dfrac{1}{(n+1)!}$, and use it to find S and the true value of $S - S_5$. What fraction of the bound in (b) does the true error use?

5. Suppose $a_n > 0$ for every n and $\sqrt[n]{a_n} \to \dfrac{1}{3}$.

(a) Decide whether each series converges, naming the test: $\sum a_n^2$, $\sum 2^n a_n$, $\sum 4^n a_n$, $\sum n^5 a_n$.

(b) Now suppose instead that $\sqrt[n]{a_n} \to 1$. Give one such sequence a_n for which $\sum a_n^2$ converges and one for which it diverges. What does this show?

6. Consider the series below, with sum S and Nth partial sum S_N.

$$\sum_{n=1}^{\infty} \frac{2 + (-1)^n}{4^n} = \frac{1}{4} + \frac{3}{16} + \frac{1}{64} + \frac{3}{256} + \cdots$$

(a) Show that $\dfrac{a_{n+1}}{a_n}$ has no limit, so the ratio test in its limit form does not apply.

(b) Show that the series still converges, and name the result you use.

(c) Use the ratio tail bound with $N = 4$ to bound $S - S_4$. Then find S exactly and compare the true error with the bound.

(d) What does the root test give for this series?

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