Calculus 2 Unit 6 worksheet: The common Maclaurin series

Name _____ Date _____ Period _____

1. Find the exact sum of each series by matching it with a series in the table. Name the function and the value of x you use. If a series diverges, say why.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\,2^n}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,2^n}{n}$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n - 1}{n!}$

2. Let $f(x) = \sin x + \cos x$.

(a) Find $f^{(k)}(0)$ for $k = 0, 1, 2, \ldots, 7$ and describe the pattern.

(b) Write the Maclaurin series of f through the x^7 term.

(c) Use the ratio test to find the radius of convergence of the series.

(d) Add the series for $\sin x$ and $\cos x$ from the table and compare with (b). Which result says the two must agree?

3. Let $f(x) = e^{-x^2/4}$.

(a) Build the Maclaurin series of f from the series for e^u. Give it in sigma notation and through its first three nonzero terms, and say for which x it equals $f(x)$.

(b) Without differentiating, find $f^{(4)}(0)$ and $f^{(5)}(0)$.

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4. Let $f(x) = \sin^2 x$.

(a) Use the identity $\sin^2 x = \dfrac{1 - \cos 2x}{2}$ and the table to find the Maclaurin series of f in sigma notation and through the x^8 term. Say for which x it equals $f(x)$.

(b) Multiply the series for $\sin x$ by itself through the x^6 term and compare with (a).

(c) Find $f^{(8)}(0)$ and $f^{(9)}(0)$.

5. For each function, build the Maclaurin series from the table. Give it in sigma notation and through its first three nonzero terms, and find the exact set of x where the series equals the function.

(a) $x \sin(x^2)$

(b) $\ln(1 + 2x^2)$

(c) $\arctan\left(\dfrac{x^3}{8}\right)$

6. Decide whether each function has a Maclaurin series. If it does not, say which derivative at 0 fails to exist. If it does, find the series and say for which x it equals the function.

(a) $f(x) = e^{|x|}$

(b) $f(x) = \dfrac{e^x - 1}{x}$ for $x \neq 0$, with $f(0) = 1$. Also find $f'''(0)$.

(c) $g(x) = \cos|x|$

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