Discrete Mathematics and Proof Unit 1 practice: Building a truth table for a compound statement (answer key)

ANSWERS AT A GLANCE

1. (a) T, F, T, T (b) F, F, F, T (c) F, T, T, T (d) F, F, F, T (e) F, T, T, T (f) T, F, T, F

2. (a) F, T, F, F (b) F, T, F, F (c) T, T, T, F (d) F, F, F, T (e) T, F, T, F (f) T, F, F, T

3. (a) F, T, F, T, F, T, F, F (b) T, T, T, T, T, F, F, F (c) F, T, F, T, F, F, F, F (d) T, F, F, F, T, T, F, T

4. (a) $C \wedge (G \vee \neg B)$; true in rows 1, 2 and 4 (b) $(H \wedge \neg V) \vee (U \wedge V)$; true in rows 1, 2, 4 and 5

WORKED SOLUTIONS

1. Give the last column of each statement's truth table, rows in the fixed order. Show a column for each piece.

(a) $\neg(P \wedge \neg Q)$ (b) $\neg P \wedge \neg Q$ (c) $\neg Q \vee \neg P$

(d) $\neg(P \vee Q)$ (e) $\neg(Q \wedge P)$ (f) $(P \vee Q) \wedge Q$

ANSWER

(a) T, F, T, T (b) F, F, F, T (c) F, T, T, T (d) F, F, F, T (e) F, T, T, T (f) T, F, T, F

SOLUTION

Rows P, Q = TT, TF, FT, FF.

(a) $\neg Q$: F, T, F, T; $P \wedge \neg Q$: F, T, F, F; last column T, F, T, T.

(b) $\neg P$: F, F, T, T; $\neg Q$: F, T, F, T; last column F, F, F, T.

(c) $\neg Q$: F, T, F, T; $\neg P$: F, F, T, T; last column F, T, T, T.

(d) $P \vee Q$: T, T, T, F; last column F, F, F, T.

(e) $Q \wedge P$: T, F, F, F; last column F, T, T, T.

(f) $P \vee Q$: T, T, T, F; last column T, F, T, F.

2. Each statement needs two or more columns before its last. Build them, inside out, and give the last column.

(a) $\neg(\neg P \vee Q)$ (b) $(\neg P \vee \neg Q) \wedge P$ (c) $(\neg P \wedge Q) \vee P$

(d) $(P \vee \neg Q) \wedge \neg P$ (e) $(P \vee Q) \wedge \neg(P \wedge \neg Q)$ (f) $\neg(P \vee Q) \vee (P \wedge Q)$

ANSWER

(a) F, T, F, F (b) F, T, F, F (c) T, T, T, F (d) F, F, F, T (e) T, F, T, F (f) T, F, F, T

SOLUTION

Rows P, Q = TT, TF, FT, FF; each column is read off the columns before it.

(a) $\neg P$: F, F, T, T; $\neg P \vee Q$: T, F, T, T; last column F, T, F, F.

(b) $\neg P$: F, F, T, T; $\neg Q$: F, T, F, T; $\neg P \vee \neg Q$: F, T, T, T; last column F, T, F, F.

(c) $\neg P$: F, F, T, T; $\neg P \wedge Q$: F, F, T, F; last column T, T, T, F.

(d) $\neg Q$: F, T, F, T; $P \vee \neg Q$: T, T, F, T; $\neg P$: F, F, T, T; last column F, F, F, T.

(e) $P \vee Q$: T, T, T, F; $\neg Q$: F, T, F, T; $P \wedge \neg Q$: F, T, F, F; $\neg(P \wedge \neg Q)$: T, F, T, T; last column T, F, T, F.

(f) $P \vee Q$: T, T, T, F; $\neg(P \vee Q)$: F, F, F, T; $P \wedge Q$: T, F, F, F; last column T, F, F, T.

3. Build each truth table on the three letters P, Q, R (eight rows, fixed order) and give its last column. A piece used twice gets one column.

(a) $(P \vee Q) \wedge \neg R$ (b) $(Q \wedge R) \vee (P \wedge \neg(Q \wedge R))$

(c) $P \wedge \neg(P \wedge R)$ (d) $(\neg P \vee R) \wedge (Q \vee \neg R)$

ANSWER

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(a) F, T, F, T, F, T, F, F (b) T, T, T, T, T, F, F, F (c) F, T, F, T, F, F, F, F (d) T, F, F, F, T, T, F, T

SOLUTION

Rows P, Q, R = TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF.

(a) $P \vee Q$: T, T, T, T, T, T, F, F; $\neg R$: F, T, F, T, F, T, F, T; last column F, T, F, T, F, T, F, F.

(b) $Q \wedge R$: T, F, F, F, T, F, F, F; $\neg(Q \wedge R)$: F, T, T, T, F, T, T, T; $P \wedge \neg(Q \wedge R)$: F, T, T, T, F, F, F, F; last column T, T, T, T, T, F, F, F. The repeated piece is built once and read twice.

(c) $P \wedge R$: T, F, T, F, F, F, F, F; $\neg(P \wedge R)$: F, T, F, T, T, T, T, T; last column F, T, F, T, F, F, F, F. One letter is unused, so two rows that differ only in that letter get the same entry.

(d) $\neg P$: F, F, F, F, T, T, T, T; $\neg P \vee R$: T, F, T, F, T, T, T, T; $\neg R$: F, T, F, T, F, T, F, T; $Q \vee \neg R$: T, T, F, T, T, T, F, T; last column T, F, F, F, T, T, F, T.

4. Write each rule in symbols, then build its truth table.

(a) Kayaks may go out on the lake when the water is calm and either a guide is on duty or the paddler is not a beginner. Let C be "the water is calm", G be "a guide is on duty" and B be "the paddler is a beginner". Use the letters in the order C, G, B and list the rows in which a kayak may go out.

(b) A greenhouse fan runs when it is hot inside and the roof vent is closed, or when it is humid inside and the roof vent is open. Let H be "it is hot inside", U be "it is humid inside" and V be "the roof vent is open". Use the letters in the order H, U, V and list the rows in which the fan runs.

ANSWER

(a) $C \wedge (G \vee \neg B)$; true in rows 1, 2 and 4 (b) $(H \wedge \neg V) \vee (U \wedge V)$; true in rows 1, 2, 4 and 5

SOLUTION

(a) $C \wedge (G \vee \neg B)$. $\neg B$: F, T, F, T, F, T, F, T; $G \vee \neg B$: T, T, F, T, T, T, F, T; last column T, T, F, T, F, F, F, F. So a kayak may go out in rows 1, 2 and 4.

(b) $(H \wedge \neg V) \vee (U \wedge V)$. $\neg V$: F, T, F, T, F, T, F, T; $H \wedge \neg V$: F, T, F, T, F, F, F, F; $U \wedge V$: T, F, F, F, T, F, F, F; last column T, T, F, T, T, F, F, F. So the fan runs in rows 1, 2, 4 and 5.

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