Discrete Mathematics and Proof Unit 1 practice: Saying a conditional without if-then (answer key)

ANSWERS AT A GLANCE

1. (a) $\neg A \vee C$ (b) $E \vee G$ (c) $\neg C \vee \neg G$ (d) $A \vee \neg E$ (e) $\neg G \vee A$ (f) $\neg E \vee C$

2. (a) $A \wedge \neg D$ (b) $\neg A \wedge \neg E$ (c) $D \wedge E$ (d) $\neg E \wedge A$ (e) $A \wedge D \wedge \neg E$ (f) $A \wedge (\neg D \vee \neg E)$

3. (a) $\neg K \to L$ (b) L false, M false (c) $A \wedge \neg B$ (d) n is a multiple of 10 and of 15, and n is not a multiple of 150; $n = 30$

4. (a) A rider is 16 or older, or the rider wears a helmet. False for 4 riders. (b) The tide is not high, or the path is under water. Negation: The tide is high and the path is not under water.

WORKED SOLUTIONS

1. Write each conditional without an arrow, as an or-statement, and drop any $\neg\neg$.

(a) $A \to C$ (b) $\neg E \to G$ (c) $C \to \neg G$

(d) $\neg A \to \neg E$ (e) $G \to A$ (f) $E \to C$

ANSWER

(a) $\neg A \vee C$ (b) $E \vee G$ (c) $\neg C \vee \neg G$ (d) $A \vee \neg E$ (e) $\neg G \vee A$ (f) $\neg E \vee C$

SOLUTION

$A \to B \equiv \neg A \vee B$: negate the hypothesis, keep the conclusion, join them with or.

(a) $\neg A \vee C$.

(b) $\neg\neg E \vee G$, which is $E \vee G$.

(c) $\neg C \vee \neg G$.

(d) $\neg\neg A \vee \neg E$, which is $A \vee \neg E$.

(e) $\neg G \vee A$.

(f) $\neg E \vee C$.

2. Negate each conditional. Write the answer as an and-statement, with every $\neg$ on a single letter.

(a) $A \to D$ (b) $\neg A \to E$ (c) $D \to \neg E$

(d) $\neg E \to \neg A$ (e) $(A \wedge D) \to E$ (f) $A \to (D \wedge E)$

ANSWER

(a) $A \wedge \neg D$ (b) $\neg A \wedge \neg E$ (c) $D \wedge E$ (d) $\neg E \wedge A$ (e) $A \wedge D \wedge \neg E$ (f) $A \wedge (\neg D \vee \neg E)$

SOLUTION

$\neg(A \to B) \equiv A \wedge \neg B$: keep the hypothesis, negate the conclusion, join them with and.

(a) $A \wedge \neg D$.

(b) $\neg A \wedge \neg E$.

(c) $D \wedge \neg\neg E$, which is $D \wedge E$.

(d) $\neg E \wedge \neg\neg A$, which is $\neg E \wedge A$.

(e) $(A \wedge D) \wedge \neg E$, which is $A \wedge D \wedge \neg E$.

(f) $A \wedge \neg(D \wedge E)$, and the De Morgan law gives $A \wedge (\neg D \vee \neg E)$.

3. Answer each one. Write statements in symbols, with every $\neg$ on a single letter.

(a) Write $K \vee L$ as a conditional. (b) Which values of L and M make $\neg L \to M$ false?

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(c) Lily says the negation of $A \to B$ is $A \to \neg B$. Find the mistake and write the correct negation.

(d) For whole numbers n: "if n is a multiple of 10 and of 15, then n is a multiple of 150." Write the and-statement a counterexample satisfies, and give the smallest positive counterexample.

ANSWER

(a) $\neg K \to L$ (b) L false, M false (c) $A \wedge \neg B$ (d) n is a multiple of 10 and of 15, and n is not a multiple of 150; $n = 30$

SOLUTION

(a) $A \vee B$ is the or-form of $\neg A \to B$, and a double negation cancels: $\neg K \to L$. Its contrapositive, starting from the other part, is right too.

(b) A conditional is false only when its hypothesis $\neg L$ is true and its conclusion M is false: L false, M false.

(c) Lily wrote another conditional, and both can be true at once, when A is false. Keep the hypothesis, negate the conclusion, join them with and: $A \wedge \neg B$.

(d) A counterexample makes the hypothesis true and the conclusion false. The common multiples of 10 and 15 are the multiples of 30, and 30 is not a multiple of 150.

4. (a) A bike rental shop has the rule: "If a rider is under 16, the rider wears a helmet." On Sunday 10 riders under 16 rented bikes, and 6 of them wore a helmet; 2 riders aged 16 or older had no helmet. Write the rule as an or-sentence. For how many riders is your or-sentence false?

(b) Hannah says: "If the tide is high, the path is under water." Write this as an or-sentence, then write its negation as an and-sentence.

ANSWER

(a) A rider is 16 or older, or the rider wears a helmet. False for 4 riders. (b) The tide is not high, or the path is under water. Negation: The tide is high and the path is not under water.

SOLUTION

(a) Negate the hypothesis and keep the conclusion: "a rider is 16 or older, or the rider wears a helmet". It is false only when both parts fail, under 16 with no helmet: 10 − 6 = 4 riders. The 2 older riders make the first part true.

(b) The or-form negates the hypothesis and keeps the conclusion: The tide is not high, or the path is under water. The negation keeps the hypothesis and negates the conclusion: The tide is high and the path is not under water.

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