Discrete Mathematics and Proof Unit 1 practice: And and or: conjunction and disjunction (answer key)

ANSWERS AT A GLANCE

1. (a) true (b) false (c) false (d) true (e) true (f) false

2. (a) true (b) true (c) false (d) true (e) false (f) true

3. (a) 6 integers (b) $x = -3$ (c) $x = -4$, $x = 4$ or $x = 9$ (d) F, F, F, T

4. (a) 13 dates (b) 32 ages (c) 9 lockers

WORKED SOLUTIONS

1. Decide whether each statement is true or false.

(a) 22 > 12 and 22 is even

(b) 38 is odd or 38 > 47

(c) 18 is a multiple of 9 and 18 is odd

(d) 36 is a perfect square or 36 < 10

(e) 15 + 21 = 36 and 15 < 21

(f) 6 × 11 = 68 or 6 > 11

ANSWER

(a) true (b) false (c) false (d) true (e) true (f) false

SOLUTION

(a) 22 > 12 is true and 22 is even, so both parts hold: **true**.

(b) 38 is even, so the first part is false, and 38 < 47, so the second part is false too. Both parts false: **false**.

(c) 18 = 9 × 2 is a multiple of 9, but 18 is even, so the second part is false: **false**.

(d) $36 = 6^2$ is a perfect square, so the or is **true**; the false second part does not matter.

(e) 15 + 21 = 36 is true and 15 < 21 is true: **true**.

(f) 6 × 11 = 66, not 68, so the first part is false, and 6 < 11, so the second is false too: **false**.

2. Take P to be true, Q true and R false. Is each statement true or false?

(a) $P \vee R$

(b) $Q \wedge \neg R$

(c) $\neg P \vee \neg Q$

(d) $(P \vee R) \wedge Q$

(e) $\neg P \wedge (Q \vee R)$

(f) $(P \wedge \neg Q) \vee (Q \wedge \neg R)$

ANSWER

(a) true (b) true (c) false (d) true (e) false (f) true

SOLUTION

First the negations: $\neg P$ is false, $\neg Q$ is false and $\neg R$ is true.

(a) P is true and R is false; an or needs one true part, so $P \vee R$ is **true**.

(b) Q is true and $\neg R$ is true; an and needs both, so it is **true**.

(c) $\neg P$ is false and $\neg Q$ is false, so the or is **false**.

(d) $P \vee R$ is true, from (a), and Q is true, so the and is **true**.

(e) $\neg P$ is false and $Q \vee R$ is true, so the and is **false**.

(f) $P \wedge \neg Q$ is false and $Q \wedge \neg R$ is true, from (b), so the or is **true**.

3. (a) How many integers x make "$x > -5$ and $x \leq 7$ and x is odd" true?

(b) Find every real number x for which "$x^2 = 9$ and $x < 0$" is true.

(c) Find every real number x for which "$x^2 = 16$ or $2x = 18$" is true.

(d) Write the column of $\neg(P \vee Q)$, top to bottom, in the usual row order.

ANSWER

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(a) 6 integers (b) $x = -3$ (c) $x = -4$, $x = 4$ or $x = 9$ (d) F, F, F, T

SOLUTION

(a) An and needs all three parts. The first two keep the integers from −4 to 7; the odd ones among them run from −3 to 7 in steps of 2: $\frac{7-(-3)}{2} + 1 = 6$ integers.

(b) $x^2 = 9$ gives $x = 3$ or $x = -3$. The and also needs $x < 0$, which keeps only $x = -3$.

(c) An or is true where at least one part is, so take the solutions of each part together. $x^2 = 16$ gives $x = 4$ or $x = -4$, and $2x = 18$ gives $x = 9$. So $x = -4$, $x = 4$ or $x = 9$.

(d) $P \vee Q$ has the column T, T, T, F, and $\neg$ flips each entry: F, F, F, T.

4. (a) A coffee shop gives free refills on every date of the month that is a multiple of 3 or a multiple of 7. How many dates of a 30-day month get free refills?

(b) A museum lets a visitor in free when the visitor is under 6 or at least 64 years old. Of the whole-number ages from 1 to 90, how many get in free?

(c) Isaac's school numbers its lockers 1 to 36, and Isaac may pick any locker whose number is even and greater than 18. How many lockers can Isaac pick from?

ANSWER

(a) 13 dates (b) 32 ages (c) 9 lockers

SOLUTION

(a) Multiples of 3: 10 dates. Multiples of 7: 4 dates. Both (multiples of 21): 1, counted twice. An or counts each date once: 10 + 4 − 1 = 13 dates.

(b) Under 6: ages 1 to 5, which is 5 ages. At least 64: ages 64 to 90, which is 27 ages. No age is in both, so the or counts 5 + 27 = 32 ages.

(c) There are 18 even numbers from 1 to 36, and 9 of them are 18 or less. Both parts hold for the other 18 − 9 = 9 lockers.

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