Discrete Mathematics and Proof Unit 1 practice: A conditional and its contrapositive (answer key)

ANSWERS AT A GLANCE

1. (a) $\neg K \to \neg J$ (b) $M \to \neg L$ (c) $\neg L \to K$ (d) $J \to M$ (e) $\neg J \to \neg L$ (f) $M \to \neg K$
2. (a) the claim and its contrapositive (b) the converse and the inverse (c) all four (d) none of the four
3. (a) $S \to \neg R$ (b) converse $\neg Q \to P$ true, inverse $\neg P \to Q$ true, contrapositive $Q \to \neg P$ true (c) $K \to (\neg M \wedge \neg N)$ (d) $B \to \neg A$
4. (a) 2 swimmers break the contrapositive and 4 break the converse. (b) $t \leq 650$ and the door is closed.

WORKED SOLUTIONS

1. Give the contrapositive of each conditional, in symbols. Cancel any double negation.

(a) $J \to K$ (b) $L \to \neg M$ (c) $\neg K \to L$

(d) $\neg M \to \neg J$ (e) $L \to J$ (f) $K \to \neg M$

ANSWER

(a) $\neg K \to \neg J$ (b) $M \to \neg L$ (c) $\neg L \to K$ (d) $J \to M$ (e) $\neg J \to \neg L$ (f) $M \to \neg K$

SOLUTION

Swap the two parts and negate both; a $\neg\neg$ cancels.

(a) $\neg K \to \neg J$.

(b) $\neg\neg M \to \neg L$, which is $M \to \neg L$.

(c) $\neg L \to \neg\neg K$, which is $\neg L \to K$.

(d) $\neg\neg J \to \neg\neg M$, which is $J \to M$.

(e) $\neg J \to \neg L$.

(f) $\neg\neg M \to \neg K$, which is $M \to \neg K$.

2. Decide which of the four are true: the claim, its converse, its inverse and its contrapositive.

(a) For integers n: if n is a multiple of 15, then n is a multiple of 5.

(b) For real x: if $x < 9$, then $x < 7$.

(c) For real x: if $6x > -36$, then $x > -6$.

(d) For integers n: if n is even, then n is a multiple of 9.

ANSWER

(a) the claim and its contrapositive (b) the converse and the inverse (c) all four (d) none of the four

SOLUTION

(a) Every multiple of 15 is a multiple of 5, so the claim and its contrapositive are true. $n = 5$ breaks the converse, and so the inverse.

(b) $x = 7$ is less than 9 but not less than 7, so the claim and its contrapositive are false. Every $x < 7$ is less than 9, so the converse and the inverse are true.

(c) Dividing by 6 > 0 keeps the inequality, so $6x > -36$ exactly when $x > -6$. The claim and the converse are true, and so are their contrapositives (the inverse is the contrapositive of the converse).

(d) $n = 2$ is even but not a multiple of 9, so the claim and its contrapositive are false. $n = 9$ is a multiple of 9 but odd, so the converse and the inverse are false.

3. Answer each in symbols, or with true and false.

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(a) The contrapositive of a conditional is $R \to \neg S$. Write the conditional.

(b) Take P true and Q false. Find the truth values of the converse, the inverse and the contrapositive of $P \to \neg Q$.

(c) Jack says the contrapositive of $(M \vee N) \to \neg K$ is $K \to (\neg M \vee \neg N)$. Find the mistake and write the correct contrapositive.

(d) The inverse of a conditional is $\neg A \to B$. Write the contrapositive of that conditional.

ANSWER

(a) $S \to \neg R$ (b) converse $\neg Q \to P$ true, inverse $\neg P \to Q$ true, contrapositive $Q \to \neg P$ true (c) $K \to (\neg M \wedge \neg N)$ (d) $B \to \neg A$

SOLUTION

(a) Taking the contrapositive twice gives back the conditional, so take the contrapositive of $R \to \neg S$: $S \to \neg R$.

(b) The converse, inverse and contrapositive of $P \to \neg Q$ are $\neg Q \to P$, $\neg P \to Q$ and $Q \to \neg P$. The converse has a true conclusion P, and the inverse and the contrapositive have false hypotheses ($\neg P$, then Q). All three are true.

(c) Jack negated $M \vee N$ as $\neg M \vee \neg N$, but by the De Morgan laws the or must turn into an and. Swap the parts and negate both: $K \to (\neg M \wedge \neg N)$.

(d) Negate both parts of the inverse to get the conditional, $A \to \neg B$. Its contrapositive is $B \to \neg A$, the inverse with its parts swapped.

4. (a) A swim club's rule says: "If a swimmer is in the deep lane, the swimmer wears a red cap." On Saturday 11 swimmers used the deep lane, 2 of them without a red cap, and 13 swimmers in all wore red caps. How many swimmers break the rule's contrapositive, and how many break its converse?

(b) A kiln's warning light follows the rule: "If the temperature is above 650 degrees or the door is open, the light is on." The light is off. What do you know about the temperature t and the door?

ANSWER

(a) 2 swimmers break the contrapositive and 4 break the converse. (b) $t \leq 650$ and the door is closed.

SOLUTION

(a) The contrapositive, "if a swimmer has no red cap, the swimmer is not in the deep lane", is broken by deep-lane swimmers without a red cap: 2. The converse, "if a swimmer wears a red cap, the swimmer is in the deep lane", is broken by red caps outside the deep lane: 9 deep-lane swimmers wore red caps, so $13 - 9 = 4$.

(b) Contrapositive: if the light is off, then the temperature is not above 650 and the door is not open (the De Morgan laws turn the or into an and). So $t \leq 650$ and the door is closed.

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