Discrete Mathematics and Proof Unit 1 practice: De Morgan's laws for statements (answer key)

ANSWERS AT A GLANCE

1. (a) $\neg A \wedge \neg B$ (b) $\neg H \vee K$ (c) $A \vee B$ (d) $A \vee \neg B \vee \neg C$ (e) $\neg A \wedge \neg B \wedge C$ (f) $A \wedge B \wedge \neg C$
2. (a) $x < -7$ or $x \geq 12$ (b) $x > -11$ and $x \leq 8$ (c) $x < -14$ or $x \geq 4$ (d) $x \leq 6$ and $y > 7$ (e) $x = 4$ or $y < 5$ (f) $x \leq 1$ or $y \leq 4$ or $x + y \geq 12$
3. (a) $\neg P \wedge (\neg Q \vee R)$ (b) $P \wedge \neg Q \wedge \neg R$: the $\neg$ never reached R (c) 56 is not a multiple of 7 or 56 is a multiple of 8; the negation is true
4. (a) $h < 105$ or $a < 8$ (b) $d \leq 10$ and $c \geq -13$ (c) 72 students

WORKED SOLUTIONS

1. Push the $\neg$ inside each statement with a De Morgan law, and cancel any double negation.

(a) $\neg(A \vee B)$ (b) $\neg(H \wedge \neg K)$ (c) $\neg(\neg A \wedge \neg B)$

(d) $\neg(\neg A \wedge B \wedge C)$ (e) $\neg(A \vee B \vee \neg C)$ (f) $\neg(\neg A \vee \neg B \vee C)$

ANSWER

(a) $\neg A \wedge \neg B$ (b) $\neg H \vee K$ (c) $A \vee B$ (d) $A \vee \neg B \vee \neg C$ (e) $\neg A \wedge \neg B \wedge C$ (f) $A \wedge B \wedge \neg C$

SOLUTION

(a) The "or" law flips $\vee$ to $\wedge$ and puts a $\neg$ on each part: $\neg A \wedge \neg B$.

(b) The "and" law gives $\neg H \vee \neg\neg K$, and $\neg\neg K$ is K: $\neg H \vee K$.

(c) The "and" law gives $\neg\neg A \vee \neg\neg B$, and both double negations cancel: $A \vee B$.

(d) The "and" law for three parts gives $\neg\neg A \vee \neg B \vee \neg C$, and $\neg\neg A$ is A: $A \vee \neg B \vee \neg C$.

(e) The "or" law for three parts gives $\neg A \wedge \neg B \wedge \neg\neg C$, and $\neg\neg C$ is C: $\neg A \wedge \neg B \wedge C$.

(f) The "or" law for three parts gives $\neg\neg A \wedge \neg\neg B \wedge \neg C$; drop the double negations: $A \wedge B \wedge \neg C$.

2. Here x and y are real numbers. Use a De Morgan law to write the negation of each condition without the word "not".

(a) $x \geq -7$ and $x < 12$ (b) $x \leq -11$ or $x > 8$

(c) $-14 \leq x < 4$ (d) $x > 6$ or $y \leq 7$

(e) $x \neq 4$ and $y \geq 5$ (f) $x > 1$ and $y > 4$ and $x + y < 12$

ANSWER

(a) $x < -7$ or $x \geq 12$ (b) $x > -11$ and $x \leq 8$ (c) $x < -14$ or $x \geq 4$ (d) $x \leq 6$ and $y > 7$ (e) $x = 4$ or $y < 5$ (f) $x \leq 1$ or $y \leq 4$ or $x + y \geq 12$

SOLUTION

(a) An "and" negates to an "or" of the negations, and each inequality flips with the other strictness: $x < -7$ or $x \geq 12$.

(b) An "or" negates to an "and" of the negations: $x > -11$ and $x \leq 8$.

(c) The chain says $x \geq -14$ and $x < 4$, so its negation is an "or": $x < -14$ or $x \geq 4$.

(d) An "or" negates to an "and": $x \leq 6$ and $y > 7$.

(e) An "and" negates to an "or", and the negation of $x \neq 4$ is $x = 4$: $x = 4$ or $y < 5$.

(f) A three-part "and" negates to a three-part "or": $x \leq 1$ or $y \leq 4$ or $x + y \geq 12$.

3. Each problem needs a De Morgan law and one more step.

(a) Rewrite $\neg\big(P \vee (Q \wedge \neg R)\big)$ so that every $\neg$ sits on a single letter.

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(b) Hannah rewrote $\neg(\neg P \vee Q \vee R)$ as $P \wedge \neg Q \wedge R$. Find the mistake and write the correct statement.

(c) Write the negation of "56 is a multiple of 7 and 56 is not a multiple of 8" as a sentence, and say which of the two is true.

ANSWER

(a) $\neg P \wedge (\neg Q \vee R)$ (b) $P \wedge \neg Q \wedge \neg R$: the $\neg$ never reached R (c) 56 is not a multiple of 7 or 56 is a multiple of 8; the negation is true

SOLUTION

(a) The "or" law on the outside gives $\neg P \wedge \neg(Q \wedge \neg R)$. The "and" law on the inner part gives $\neg Q \vee \neg\neg R$, which is $\neg Q \vee R$. So the answer is $\neg P \wedge (\neg Q \vee R)$.

(b) The "or" law puts a $\neg$ on every part of the chain, so R must become $\neg R$. The correct statement is $P \wedge \neg Q \wedge \neg R$.

(c) $56 = 7 \times 8$ is a multiple of 7, but also of 8, so the second part fails and the original "and" is false. By the "and" law, with the double negation cancelled, the negation is "56 is not a multiple of 7 or 56 is a multiple of 8", which is true since its second part is.

4. (a) A fair ride lets a rider on when the rider's height h is at least 105 cm and the rider's age a is at least 8 years. Write the condition for a rider to be turned away as an "or" of two inequalities.

(b) A mountain school stays closed when the snow depth d is more than 10 cm or the temperature c is below -13 degrees Celsius. Write the condition for the school to open as an "and" of two inequalities.

(c) In Hannah's grade of 137 students, 65 play a sport or play an instrument (or both). How many students play neither a sport nor an instrument?

ANSWER

(a) $h < 105$ or $a < 8$ (b) $d \leq 10$ and $c \geq -13$ (c) 72 students

SOLUTION

(a) Turned away is the negation of the "and", so it is the "or" of the two negations: $h < 105$ or $a < 8$.

(b) Open is the negation of the "or", so it is the "and" of the two negations: $d \leq 10$ and $c \geq -13$.

(c) With S for "plays a sport" and I for "plays an instrument", neither is $\neg S \wedge \neg I$, which the "or" law says is $\neg(S \vee I)$: everyone outside the 65. That is $137 - 65 = 72$ students.

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