# Discrete Mathematics and Proof Unit 1 practice: Exclusive or and logic circuits (answer key)

**ANSWERS AT A GLANCE**

**1.** (a) 1 (b) 0 (c) 1 (d) 1 (e) 0 (f) 1

**2.** (a) F (b) T (c) $Q$ (d) $P$ (e) $\neg P$ (f) $\neg P$

**3.** (a) $Y = (\neg P \wedge Q) \oplus R$ (b) $Y = 1$ (c) rows 1, 3, 6 and 7

**4.** (a) 00101110 (b) Yes: it has 4 ones, an even number (c) $V = C \wedge (L \vee R)$; 2 gates

**WORKED SOLUTIONS**

**1.** Find each value. A bit stands for a truth value: 1 for T and 0 for F.

(a) $0 \oplus 1$ (b) $0 \oplus 1 \oplus 1$ (c) $0 \oplus 1 \oplus 1 \oplus 1$

(d) $(1 \oplus 0) \wedge 1$ (e) $0 \vee (1 \oplus 1)$ (f) $0 \oplus 1 \oplus 1 \oplus 0 \oplus 1$

**ANSWER**

(a) 1 (b) 0 (c) 1 (d) 1 (e) 0 (f) 1

**SOLUTION**

(a) $0 \oplus 1 = 1$: the result is 1 exactly when the two bits differ.

(b) 2 of the bits are 1. A chain of $\oplus$ is 1 exactly when an odd number of its bits are 1, so it is 0.

(c) 3 of the bits are 1, so the chain is 1.

(d) $1 \oplus 0 = 1$, then $1 \wedge 1 = 1$.

(e) $1 \oplus 1 = 0$, then $0 \vee 0 = 0$.

(f) 3 of the bits are 1, so the chain is 1.

**2.** Use the laws of exclusive or to reduce each statement to one of $P$, $\neg P$, $Q$, $\neg Q$, T or F.

(a) $P \oplus \mathrm{F} \oplus P$ (b) $P \oplus (\mathrm{T} \oplus P)$

(c) $\neg Q \oplus \mathrm{T}$ (d) $(P \oplus Q \oplus R) \oplus (R \oplus Q)$

(e) $\neg(P \oplus Q) \oplus Q$ (f) $(Q \oplus R) \oplus \neg(P \oplus R) \oplus Q$

**ANSWER**

(a) F (b) T (c) $Q$ (d) $P$ (e) $\neg P$ (f) $\neg P$

**SOLUTION**

(a) Regroup: $(P \oplus P) \oplus \mathrm{F} \equiv \mathrm{F} \oplus \mathrm{F} \equiv \mathrm{F}$.

(b) Regroup: $(P \oplus P) \oplus \mathrm{T} \equiv \mathrm{F} \oplus \mathrm{T} \equiv \mathrm{T}$.

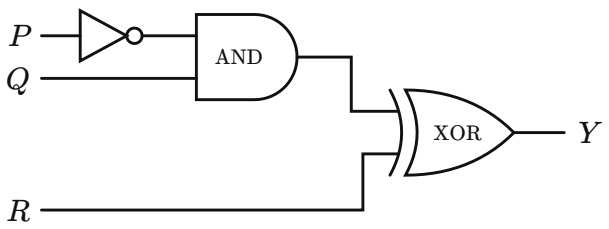
(c) Exclusive or with T negates: $\neg Q \oplus \mathrm{T} \equiv \neg\neg Q \equiv Q$.

(d) Regroup: $P \oplus (Q \oplus Q) \oplus (R \oplus R) \equiv P \oplus \mathrm{F} \oplus \mathrm{F} \equiv P$.

(e) $\neg(P \oplus Q) \equiv P \oplus Q \oplus \mathrm{T}$, so this is $P \oplus (Q \oplus Q) \oplus \mathrm{T} \equiv P \oplus \mathrm{T} \equiv \neg P$.

(f) Write $\neg(P \oplus R) \equiv P \oplus R \oplus \mathrm{T}$. Then $Q$ and $R$ each appear twice and cancel, leaving $P \oplus \mathrm{T} \equiv \neg P$.

**3.** The circuit has inputs $P$, $Q$, $R$ and output $Y$. Rows are numbered 1 to 8 in the fixed order $P$, $Q$, $R$ = TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF. A bit 1 is T and 0 is F.




(a) Write $Y$ as a statement in $P$, $Q$ and $R$.

(b) Find the bit $Y$ when $P = 0$, $Q = 1$ and $R = 0$.

(c) List the rows on which $Y$ is true.


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**ANSWER**

(a) $Y = (\neg P \wedge Q) \oplus R$ (b) $Y = 1$ (c) rows 1, 3, 6 and 7

**SOLUTION**

(a) Work from the inputs. The NOT gate carries $\neg P$, the AND gate is fed by $\neg P$ and $Q$, and its output and $R$ feed the XOR gate: $Y = (\neg P \wedge Q) \oplus R$.

(b) $\neg P = 1$. The AND gate on 1 and 1 gives 1, and the XOR gate on 1 and 0 gives $Y = 1$.

(c) $\neg P \wedge Q$ is T only when $P$ is F and $Q$ is T (rows 5, 6). $Y$ is T where that wire and $R$ differ: row 6 ($R$ F) and rows 1, 3, 7 ($R$ T).

**4.** (a) A weather balloon sends each 7-bit reading followed by an even parity bit. Write the 8-bit word it sends for the reading 0010111.

(b) Another word from the balloon arrives as 10100101. Does it pass the parity check?

(c) With $L$ "the left door is open", $R$ "the right door is open" and $C$ "the van is moving", a warning light $V$ is on when the van is moving and a door is open. Write $V$ as a statement in the three letters, and count the gates its circuit needs (XOR gates allowed).

**ANSWER**

(a) 00101110 (b) Yes: it has 4 ones, an even number (c) $V = C \wedge (L \vee R)$; 2 gates

**SOLUTION**

(a) The reading has 4 ones. The parity bit makes the count even, so it is 0, and the word is 00101110.

(b) A word sent with an even parity bit has an even number of 1s. Yes: it has 4 ones, an even number.

(c) $V = C \wedge (L \vee R)$. Its circuit: an OR gate on $L$ and $R$, then an AND gate with $C$, 2 gates.


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