Discrete Mathematics and Proof Unit 1 practice: Tautologies and contradictions

Name _____ Date _____ Period _____

1. Fill in the two rows of each statement's table (P = T, then P = F). Then say whether it is a tautology, a contradiction or neither.

(a) $P \leftrightarrow P$

(b) $\neg(P \vee P) \wedge P$

(c) $(P \rightarrow \neg P) \vee P$

(d) $(P \vee \neg P) \rightarrow (P \wedge \neg P)$

(e) $(P \rightarrow P) \vee \neg P$

(f) $P \leftrightarrow \neg\neg P$

2. Classify each statement as a tautology, a contradiction or neither (rows P, Q = TT, TF, FT, FF). When it is neither, name the rows where it is false.

(a) $(P \vee Q) \vee \neg P$

(b) $\neg P \vee (P \wedge Q)$

(c) $\neg Q \vee (P \rightarrow Q)$

(d) $\neg(P \vee \neg Q) \wedge \neg Q$

(e) $\neg(P \wedge Q) \vee P$

(f) $\neg\big(Q \rightarrow (P \vee Q)\big)$

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3. Each of these takes two steps. Rows run in the fixed order: P, Q = TT, TF, FT, FF, and P, Q, R = TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF.

(a) Name every row where $\big(\neg R \wedge (P \to R)\big) \to \neg(P \wedge Q)$ is false, or say there is none.

(b) Premises $P \to Q$ and $Q \to P$, conclusion P. Write the argument as one conditional and decide whether it is valid.

(c) Use the biconditional test: are $\neg P \to (P \vee Q)$ and Q logically equivalent? If not, name the rows where they disagree.

4. (a) Hannah says: "If the kettle is on and the water is hot, then the water is hot." Using K for "the kettle is on" and W for "the water is hot", write the sentence in symbols and say whether it is a tautology, a contradiction or neither.

(b) A friend reasons: "The alarm sounds if and only if the door is open. The alarm is silent. So the door is closed." Using A for "the alarm sounds" and D for "the door is open", write the argument as one conditional and decide whether it is valid.

(c) Hannah builds the truth table of a contradiction S that uses 3 letters, and adds a column for $\neg S$. How many entries of the $\neg S$ column are T?

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