Discrete Mathematics and Proof Unit 1 practice: Truth table for the conditional (answer key)

ANSWERS AT A GLANCE

1. (a) false (b) false (c) true (d) true (e) true (f) true (g) true (h) true

2. (a) T, F, T, T (b) T, T, F, T (c) F, T, F, T (d) T, F, T, T (e) T, F, T, T (f) F, F, T, F

3. (a) $Q \to \neg P$ is false (b) P is false (c) false (d) T, F, F, T, T, T, T

4. (a) row 2, broken (b) broken on 2 days, true on 18 days

WORKED SOLUTIONS

1. Is each conditional true or false? Check its hypothesis and its conclusion first.

(a) If 30 is even, then 30 is a multiple of 4.

(b) If 56 > 50, then 56 is odd.

(c) If 35 is a multiple of 5, then 35 < 40.

(d) If 50 is a perfect square, then 50 is even.

(e) If 7 + 9 = 16, then 7 < 9.

(f) If 25 is prime, then 25 + 2 is prime.

(g) If 15 is a multiple of 6, then 15 is a multiple of 4.

(h) If 4 is odd, then $4^2 > 50$.

ANSWER

(a) false (b) false (c) true (d) true (e) true (f) true (g) true (h) true

SOLUTION

A conditional is false only in row 2: true hypothesis, false conclusion.

(a) Hypothesis true, conclusion false: row 2, false.

(b) Hypothesis true, conclusion false: row 2, false.

(c) Hypothesis true, conclusion true: row 1, true.

(d) Hypothesis false, conclusion true: row 3, true.

(e) Hypothesis true, conclusion true: row 1, true.

(f) Hypothesis false, conclusion false: row 4, true.

(g) Hypothesis false, conclusion false: row 4, true.

(h) Hypothesis false, conclusion false: row 4, true.

2. Fill in the column of each statement for the rows P, Q = TT, TF, FT, FF. Use a helper column for each inner piece.

(a) $\neg Q \to \neg P$

(b) $\neg P \to \neg Q$

(c) $Q \to \neg Q$

(d) $P \to (P \wedge Q)$

(e) $(P \vee Q) \to Q$

(f) $\neg(Q \to P)$

ANSWER

(a) T, F, T, T (b) T, T, F, T (c) F, T, F, T (d) T, F, T, T (e) T, F, T, T (f) F, F, T, F

SOLUTION

An arrow gives F only where its hypothesis is T and its conclusion is F.

(a) $\neg Q$: F, T, F, T; $\neg P$: F, F, T, T; $\neg Q \to \neg P$: T, F, T, T.

(b) $\neg P$: F, F, T, T; $\neg Q$: F, T, F, T; $\neg P \to \neg Q$: T, T, F, T.

(c) $\neg Q$: F, T, F, T; $Q \to \neg Q$: F, T, F, T.

(d) $P \wedge Q$: T, F, F, F; $P \to (P \wedge Q)$: T, F, T, T.

(e) $P \vee Q$: T, T, T, F; $(P \vee Q) \to Q$: T, F, T, T.

(f) $Q \to P$: T, T, F, T; $\neg(Q \to P)$: F, F, T, F.

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3. Each of these takes two steps.

(a) $P \to \neg Q$ is false. Is $Q \to \neg P$ true or false?

(b) Both $P \to Q$ and $P \to \neg Q$ are true. Is P true or false?

(c) Find the value of $(Q \to P) \to \neg R$ when P is true, Q is false and R is true.

(d) Give the column of $P \to (Q \wedge R)$ on P, Q, R (eight rows, fixed order).

ANSWER

(a) $Q \to \neg P$ is false (b) P is false (c) false (d) T, F, F, F, T, T, T, T

SOLUTION

(a) A conditional is false only with a true hypothesis and a false conclusion, so P is true and Q is true. Then $Q \to \neg P$ is false.

(b) If P were true, Q and $\neg Q$ would both have to be true, which cannot happen. So P is false.

(c) $Q \to P$ is true, since Q is false, and $\neg R$ is false, so the outer arrow goes from true to false: false.

(d) Rows P, Q, R = TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF. $Q \wedge R$: T, F, F, F, T, F, F, F; $P \to (Q \wedge R)$: T, F, F, F, T, T, T, T.

4. (a) A bakery sign says: "If you buy at least 12 muffins, you get a free coffee." Hannah buys 18 muffins and does not get a free coffee. Which row of the table for the sign's promise is this, and was the promise broken?

(b) A school's rule says: "If it is sunny at noon, recess is outside." Over 20 school days it was sunny at noon on 15 days; recess was outside on 13 of the sunny days and on 3 of the other days. On how many days was the rule broken, and on how many was it true?

ANSWER

(a) row 2, broken (b) broken on 2 days, true on 18 days

SOLUTION

(a) Hypothesis "at least 12 muffins" is true; conclusion "free coffee" is false. That is row 2, and only row 2 breaks a promise, so it was broken.

(b) The rule breaks only on a sunny day with recess inside (row 2): 15 − 13 = 2 days. Every other day is true, including the 5 days that were not sunny (rows 3 and 4): 20 − 2 = 18 days.

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