Discrete Mathematics and Proof Unit 1 quiz 1: And and or: conjunction and disjunction, Building a truth table for a compound statement, Truth table for the conditional (answer key)

8 questions, 19 points

ANSWERS AT A GLANCE

1. (a) true (b) true
2. Last column T, T, T, F
3. True in rows 5, 6 and 8.
4. (a) A, C and E (b) E only (c) the points with $x \leq 0$ and $y \leq 0$: the third quadrant together with its edges, the origin and the negative halves of both axes.
5. Only the frost alarm went off: F is true, V and W are false. $V \vee \neg W$ is true.
6. False only in rows 6, 7 and 8.
7. Last column F, F, T, T, F, F, T, F
8. (a) No. With Q true the statement is in row 1 or row 3, both true, and P is false in row 3. (b) No smoke. With Q false only row 4 keeps $P \to Q$ true, and there P is false. (c) No. Steam and no smoke is row 3, where $P \to Q$ is true.

WORKED SOLUTIONS

1. Let $a = 28$ and $b = 19$. Decide whether each sentence is true or false. (2 points)

(a) ($a > b$ and a is even) or $a + b$ is a multiple of 5.

(b) $a > b$ and (a is even or $a + b$ is a multiple of 5).

ANSWER

(a) true (b) true

SOLUTION

First settle the three parts. 28 > 19, so "$a > b$" is true. 28 is even, so "a is even" is true. $a + b = 47$, which is not a multiple of 5, so the last part is false.

(a) Both parts of the bracket are true, so the bracket is true, and an or with a true part is **true**.

(b) The bracket is true because "a is even" is true, and $a > b$ is true, so both parts of the and hold: **true**.

The two sentences have the same three parts and differ only in the grouping. With "and" and "or" mixed, the grouping can change the value.

2. Build a truth table for $\neg(\neg P \vee Q) \vee Q$, with a column for every piece, rows in the fixed order. (2 points)

ANSWER

Last column T, T, T, F

SOLUTION

Two letters give 4 rows: P, Q = TT, TF, FT, FF. The columns, built from the inside out, each from columns already on the page: $\neg P$: F, F, T, T; $\neg P \vee Q$: T, F, T, T; $\neg(\neg P \vee Q)$: F, T, F, F. The last column, for $\neg(\neg P \vee Q) \vee Q$, is T, T, T, F.

SCORING

1 point: four rows in the fixed order and correct helper columns. 1 point: last column T, T, T, F.

3. Without writing out the whole truth table, list every row of the table for $\neg P \wedge (Q \vee \neg R)$, letters P, Q, R in the fixed order, in which the statement is true. Explain how you found them. (2 points)

ANSWER

True in rows 5, 6 and 8.

SOLUTION

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An and is true only where both parts are true. $\neg P$ is true only where P is false, which is rows 5 to 8, so no other row can be true.

In rows 5 to 8, Q is T, T, F, F and R is T, F, T, F. The or $Q \vee \neg R$ is false only where Q is false and $\neg R$ is false, that is, Q false and R true: row 7. So the statement is true in rows 5, 6 and 8.

SCORING

1 point: only rows 5 to 8 can be true, because $\neg P$ must be. 1 point: row 7 ruled out (Q false, R true), leaving rows 5, 6 and 8.

4. Consider the points $A(3, -2)$, $B(-1, -4)$, $C(0, 5)$, $D(-2, 0)$ and $E(4, 1)$ in the coordinate plane, each written as (x, y). (3 points)

(a) Which of the points make "$x > 0$ or $y > 0$" true?

(b) Which of the points make "$x > 0$ and $y > 0$" true?

(c) Describe every point (x, y) of the plane for which "$x > 0$ or $y > 0$" is false.

ANSWER

(a) A, C and E (b) E only (c) the points with $x \leq 0$ and $y \leq 0$: the third quadrant together with its edges, the origin and the negative halves of both axes.

SOLUTION

(a) An or needs one true part. A: 3 > 0, true. B: −1 > 0 and −4 > 0 are both false, so false. C: 0 > 0 is false, but 5 > 0 is true, so true. D: −2 > 0 and 0 > 0 are both false, so false. E: true. The points are A, C and E.

(b) An and needs both parts. A fails $y > 0$, B and D fail both, and C fails $x > 0$, since 0 is not greater than 0. Only E, with 4 > 0 and 1 > 0, passes.

(c) An or is false only when both parts are false. "$x > 0$" is false when $x \leq 0$, and "$y > 0$" is false when $y \leq 0$. So the sentence is false exactly at the points with $x \leq 0$ and $y \leq 0$: the third quadrant, together with the origin and the negative halves of both axes, such as D.

SCORING

1 point for each part. (a) needs A, C and E and no others. (b) needs E alone, with C ruled out because 0 > 0 is false. (c) needs "$x \leq 0$ and $y \leq 0$" (or the same region in words), reached from the rule that an or is false only when both parts are.

5. A greenhouse control panel records whether each of three alarms went off overnight. Let F be "the frost alarm went off", V be "the vent alarm went off" and W be "the water alarm went off". Mei reads two facts from the log: $\neg F \vee W$ is false, and $F \wedge V$ is false. Which alarms went off? Then find the truth value of $V \vee \neg W$. (2 points)

ANSWER

Only the frost alarm went off: F is true, V and W are false. $V \vee \neg W$ is true.

SOLUTION

A false or has both parts false, so $\neg F$ is false and W is false. Then F is true.

$F \wedge V$ is false while F is true, so the other part must be the false one: V is false.

So only the frost alarm went off: F is true, V and W are false. Then $\neg W$ is true, so $V \vee \neg W$ is true.

6. The statement $\neg P \to (Q \wedge R)$ is a conditional. Name its hypothesis and its conclusion, find the rows of the table (letters P, Q, R in the fixed order) where the hypothesis is true and where the conclusion is false, and from those find every row in which the whole statement is false. (2 points)

ANSWER

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False only in rows 6, 7 and 8.

SOLUTION

A conditional is false only where its hypothesis is true and its conclusion is false. The hypothesis is $\neg P$, true in rows 5, 6, 7 and 8. The conclusion is $Q \wedge R$, false in rows 2, 3, 4, 6, 7 and 8. Both happen only in rows 6, 7 and 8, so the statement is false there and true in every other row.

SCORING

1 point: hypothesis $\neg P$ and conclusion $Q \wedge R$ named, with the rows where each is true or false. 1 point: false only in rows 6, 7 and 8.

7. Build a truth table for $\neg(Q \vee \neg R) \vee (P \wedge \neg Q)$, with a column for every piece, rows in the fixed order P, Q, R. (3 points)

ANSWER

Last column F, F, T, T, F, F, T, F

SOLUTION

Inside out: $\neg R$, then $Q \vee \neg R$, then its negation; separately $\neg Q$, then $P \wedge \neg Q$. The last column is the or of $\neg(Q \vee \neg R)$ and $P \wedge \neg Q$.

Row	P	Q	R	$\neg R$	$Q \vee \neg R$	$\neg(Q \vee \neg R)$	$\neg Q$	$P \wedge \neg Q$	$\neg(Q \vee \neg R) \vee (P \wedge \neg Q)$
1	T	T	T	F	T	F	F	F	F
2	T	T	F	T	T	F	F	F	F
3	T	F	T	F	F	T	T	T	T
4	T	F	F	T	T	F	T	T	T
5	F	T	T	F	T	F	F	F	F
6	F	T	F	T	T	F	F	F	F
7	F	F	T	F	F	T	T	F	T
8	F	F	F	T	T	F	T	F	F

SCORING

1 point: eight rows in the fixed order, with columns for $\neg R$ and $\neg Q$. 1 point: correct columns for $Q \vee \neg R$, its negation and $P \wedge \neg Q$. 1 point: correct last column, F, F, T, T, F, F, T, F (follow-through from one earlier slip loses only that point).

8. A smoke detector's manual says: "If there is smoke in the room, the alarm sounds." Let P be "there is smoke in the room" and Q be "the alarm sounds", and suppose the manual's statement $P \to Q$ is true. Answer each part from the rows of the table for $P \to Q$. (3 points)

(a) The alarm is sounding. Must there be smoke in the room?

(b) The alarm is silent. Is there smoke in the room?

(c) One evening the alarm sounds with no smoke in the room, only steam from a shower. Does that show the manual's statement is false?

ANSWER

(a) No. With Q true the statement is in row 1 or row 3, both true, and P is false in row 3. (b) No smoke. With Q false only row 4 keeps $P \to Q$ true, and there P is false. (c) No. Steam and no smoke is row 3, where $P \to Q$ is true.

SOLUTION

(a) Q true leaves rows 1 and 3. $P \to Q$ is true in both, so neither is ruled out. P is true in row 1 and false in row 3, so the alarm alone does not show there is smoke.

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(b) Q false leaves rows 2 and 4. Row 2 is the one row where $P \to Q$ is false, and the statement is true, so the room is in row 4: P is false, there is no smoke.

(c) P is false and Q is true: row 3. There $P \to Q$ is true, so the evening breaks nothing. The manual promised what happens when there is smoke, and said nothing about what happens without it.

SCORING

1 point for each part: (a) no, rows 1 and 3 both possible; (b) no smoke, only row 4; (c) no, row 3 is true.

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