Discrete Mathematics and Proof Unit 1 quiz 2: Tautologies and contradictions, De Morgan's laws for statements, A conditional and its contrapositive

Name _____ Date _____ Period _____

8 questions, 21 points

1. Classify $(P \wedge Q) \wedge \neg(P \vee Q)$ as a tautology, a contradiction or neither, using a truth table with the letters P, Q in the fixed order. If it is neither, name every row where it is false. (2 points)

2. A school lab allows a student to use the laser cutter exactly when the student is wearing safety glasses and a teacher is in the room. (2 points)

(a) Use a De Morgan law to say, in words, exactly when a student may not use the laser cutter.

(b) A student who is not wearing safety glasses says: "A teacher is in the room, so I may use the cutter." Use your answer to (a) to explain why not.

3. Tickets in a school raffle are numbered 1 to 24. A ticket wins a small prize unless its number is a multiple of 2 or a multiple of 5. (2 points)

(a) Use a De Morgan law to write the winning condition as an "and" of two conditions, with no "not" in front of a bracket.

(b) How many tickets win a small prize?

4. Decide whether the two statements in each pair are contradictory. When a pair is not, name every row (letters P, Q in the fixed order) where the two statements have the same truth value. (3 points)

(a) $P \leftrightarrow Q$ and $(P \wedge \neg Q) \vee (\neg P \wedge Q)$

(b) $P \wedge Q$ and $\neg P \wedge \neg Q$

(c) $\neg P \rightarrow Q$ and $\neg P \wedge \neg Q$

5. For each statement, find every combination of truth values of its letters that gives the statement the value stated. (3 points)

(a) $\neg(P \vee \neg Q)$ is true.

(b) $\neg(P \wedge Q) \wedge P$ is true.

(c) $\neg(P \vee Q) \vee R$ is false. Name the rows of the table on P, Q, R (in the fixed order).

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6. Priya keeps two rules for her garden: "If the soil is dry, I water the beds" and "If I water the beds, the level in the rain barrel falls." Let D be "the soil is dry", W be "Priya waters the beds" and B be "the level in the rain barrel falls". (3 points)

(a) One evening the level has not fallen, and Priya argues from her two rules: "So the soil was not dry." Write her argument as one conditional and decide, without writing all eight rows, whether it is valid.

(b) Her neighbour Omar argues from the same two rules: "The soil was not dry. So the level did not fall." Write his argument as one conditional and decide whether it is valid. If it is not, name every counterexample row (letters D, W, B in the fixed order).

7. For each statement below, say whether it is the converse, the inverse or the contrapositive of $P \to \neg Q$ (after cancelling any double negation), or none of these. Then say which of the four statements are logically equivalent to $P \to \neg Q$.

(a) $Q \to \neg P$ (b) $\neg Q \to P$ (c) $\neg P \to Q$ (d) $\neg P \to \neg Q$ (3 points)

8. A museum checks that this is true of every visitor: "If a visitor is under 12, then the visitor is with an adult." Which of the sentences below say the same thing as this rule? For each one that does not, describe a visitor who keeps the rule but breaks the sentence. (3 points)

(a) If a visitor is not with an adult, then the visitor is 12 or older.

(b) If a visitor is with an adult, then the visitor is under 12.

(c) If a visitor is 12 or older, then the visitor is not with an adult.

(d) No visitor under 12 is without an adult.

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