Discrete Mathematics and Proof Unit 1 quiz 3: Saying a conditional without if-then, Exclusive or and logic circuits

Name _____ Date _____ Period _____

7 questions, 22 points

1. A youth soccer league keeps a spreadsheet of its players. In each row, cell B is TRUE when the player is under 14, and cell C is TRUE when a signed medical form is on file. The league's rule is: "If a player is under 14, then a signed medical form is on file." The spreadsheet has the functions AND, OR and NOT, each giving TRUE or FALSE, but no if-then function. (3 points)

(a) Write a formula using B2 and C2 that is TRUE exactly when the player in row 2 keeps the rule.

(b) Write a formula using B2 and C2 that is TRUE exactly when the player in row 2 breaks the rule.

(c) Kofi used `=OR(B2, C2)` for (a). Describe every kind of player for whom his formula gives the wrong answer.

2. Write the negation of each statement with no $\rightarrow$ in it and every $\neg$ on a single letter. (3 points)

(a) $\neg P \rightarrow \neg Q$

(b) $(P \vee Q) \rightarrow R$

(c) $P \rightarrow (Q \rightarrow R)$

(d) $(P \rightarrow Q) \rightarrow R$

Free sample from math.wiki for classroom use. Do not post. math.wiki

3. Each claim below is about every real number x. For each one, write the and-statement that a counterexample must satisfy. Then give a counterexample, or show that none exists, and say whether the claim is true. (3 points)

(a) If $x^2 > 4x$, then $x > 4$.

(b) If $x > 2$, then $x^2 > 2x$.

(c) If $x \neq 0$, then $x + \dfrac{1}{x} \geq 2$.

4. A rotary sensor on a robot arm reports each position as a 5-bit Gray code, so that neighbouring positions differ in exactly one bit. The binary number $b_1 b_2 b_3 b_4 b_5$ (b_1 on the left) has the Gray code $g_1 g_2 g_3 g_4 g_5$, where $g_1 = b_1$ and $g_i = b_{i-1} \oplus b_i$ for $i = 2, 3, 4, 5$. (3 points)

(a) Write 24 in 5-bit binary and find its Gray code.

(b) Show that $b_i = b_{i-1} \oplus g_i$ for $i = 2, 3, 4, 5$. The sensor reports the Gray code 11010: find the binary number and its value.

5. A science-fair robot's motor has four inputs: L, "the left bump sensor is pressed"; R, "the right bump sensor is pressed"; O, "the remote override is on"; B, "the battery is low". The motor runs when exactly one bump sensor is pressed, or when the override is on and the battery is not low. (3 points)

(a) Write the motor's statement M, and describe a circuit for it with as few gates as you can (XOR gates allowed). How many gates?

(b) Both bump sensors are pressed, the override is on and the battery is low. Does the motor run?

(c) The parts bin has no XOR gates. Using the four-gate form of exclusive or, how many gates does the circuit need now?

Free sample from math.wiki for classroom use. Do not post. math.wiki

6. A puzzle box has three lamps and three buttons, and all the lamps start off. Button 1 flips lamps 1 and 2, button 2 flips lamps 2 and 3, and button 3 flips lamps 1 and 3 (a flip turns a lamp that is off on, and one that is on off). Let x_1, x_2, x_3 be "button 1 (or 2, or 3) was pressed an odd number of times", and read a lamp as T when it is on. (3 points)

(a) Write the state of each lamp as a statement in x_1, x_2, x_3, and use the laws of exclusive or to explain why the order of the presses does not matter.

(b) Find every set of buttons, each pressed once, that leaves lamps 1 and 3 on and lamp 2 off.

(c) Show that no sequence of presses leaves exactly one lamp on.

7. You may use the law $(A \vee B) \wedge (A \vee C) \equiv A \vee (B \wedge C)$, the De Morgan laws, and the fact that an or with one part that is false in every row has the same truth value as its other part. (4 points)

(a) Without a truth table, show that $(P \to Q) \wedge (P \to \neg Q) \equiv \neg P$.

(b) Suppose that, starting from an assumption P, you have shown both that Q follows and that $\neg Q$ follows. Use (a) to say what you now know about P, and why.

Statements	Reasons

Free sample from math.wiki for classroom use. Do not post. math.wiki