Discrete Mathematics and Proof Unit 1 test: Statements, truth tables and logical equivalence

Name _____ Date _____ Period _____

13 questions, 41 points

1. Let $n = 69$. Let P be "n is even", Q be "$n > 50$" and R be "n is a multiple of 3". Find the truth value of each statement, and show how you decided. (3 points)

(a) $(P \vee Q) \to R$

(b) $(P \wedge \neg R) \leftrightarrow Q$

2. For which real numbers x is the sentence below true, and for which is it false? Describe each set with inequalities joined by "and" or "or".

$$(x < -4 \text{ or } x > -2) \text{ and } -8 \leq x \leq 1$$

(3 points)

3. For positive integers n, consider the claim: "if n is a multiple of 4 and a multiple of 10, then n is a multiple of 8." (3 points)

(a) Write its contrapositive as an if-then sentence. Do not use the words "not both" or "it is not the case".

(b) Find the smallest positive integer n that makes the contrapositive false, or show that no such n exists.

(c) Is the claim itself true? Answer from (b), without testing the claim again.

4. (3 points)

(a) Write $P \vee Q$ using only the connectives $\neg$ and $\to$.

(b) Write $P \wedge Q$ using only the connectives $\neg$ and $\to$.

(c) Write $P \leftrightarrow Q$ using only the connectives $\neg$ and $\vee$.

Show briefly why each of your answers is equivalent to the statement it replaces.

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5. Let $x = -1$, and let P be "$x^2 > 4$", Q be "$x > 2$" and R be "$x < 0$". (4 points)

(a) Give the truth values of P, Q and R.

(b) Give the truth values of $P \to Q$, $R \to P$ and $\neg P \to R$.

(c) Now let x be any integer, with P and R the same sentences about it. Find every integer x for which $\neg P \to R$ is false.

(d) Is there any integer x for which $Q \to P$ is false? Explain.

6. Build a truth table for $(P \to \neg Q) \leftrightarrow (Q \vee R)$, with a column for every piece, rows in the fixed order P, Q, R. Then list the rows in which the statement is true. (3 points)

7. A climbing gym's rule: a climber may climb when the climber is wearing a harness and, if the climber is under 16, a parent has signed the waiver. Let H be "the climber is wearing a harness", U be "the climber is under 16" and W be "a parent has signed the waiver". (3 points)

(a) Write the rule as a statement in H, U and W.

(b) Build its truth table, rows in the order H, U, W.

(c) Mason is 15, is wearing a harness and has no signed waiver. Leah is 17, is wearing a harness and has no signed waiver. Name each one's row and say whether each may climb.

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8. Diego's rule: "If it is a weekday, then if it is raining, I take the bus." His sister Leah writes it as: "If it is a weekday and it is raining, I take the bus." Let W be "it is a weekday", R be "it is raining" and B be "Diego takes the bus". (3 points)

(a) Write each version as a statement in W, R and B.

(b) Without writing the whole table, find every row (letters in the order W, R, B) in which each statement is false.

(c) Do the two versions ever say different things? Explain.

9. Decide whether $\big(P \wedge (Q \to R)\big) \to (P \wedge R)$ is a tautology, a contradiction or neither (letters P, Q, R in the fixed order). If it is neither, name every row where it is false. You need not write all eight rows, but your reasoning must account for every row. (3 points)

10. A delivery drone must land when its battery is below 20 percent, or when the wind speed is above 50 km/h while it is raining. Otherwise it keeps flying. (3 points)

(a) Use the De Morgan laws to write the condition for the drone to keep flying, with no "not" in front of a bracketed condition and each inequality written without "not".

(b) The drone reads a battery level of 25 percent and a wind speed of 55 km/h, and it is not raining. Give the truth value of each simple part of your condition in (a), and say whether the drone keeps flying.

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11. Use the De Morgan laws to rewrite each statement with every $\neg$ on a single letter. Then, without a full truth table, classify it as a tautology, a contradiction or neither. If it is neither, name the rows (letters P, Q in the fixed order) where it is true. (3 points)

(a) $\neg(\neg P \vee Q) \wedge \neg(P \wedge \neg Q)$

(b) $\neg(P \wedge Q) \vee \neg(\neg P \wedge \neg Q)$

(c) $\neg(P \vee Q) \vee \neg(P \vee \neg Q)$

12. A 4-bit adder adds $a = a_3a_2a_1a_0$ and $b = b_3b_2b_1b_0$ one column at a time, from the right, as you add by hand. Column i takes a_i, b_i and the carry c_i from the column to its right ($c_0 = 0$). It outputs the sum bit $s_i = a_i \oplus b_i \oplus c_i$ and passes on the carry c_{i+1}, which is 1 when at least two of a_i, b_i, c_i are 1. (3 points)

(a) Add 7 and 14 this way. Write both in 4-bit binary, and give c_1, c_2, c_3, c_4 and $s_3s_2s_1s_0$.

(b) Check that the 5-bit number $c_4s_3s_2s_1s_0$ is 7 + 14.

13. A library's rule: "A student may borrow a laptop if and only if the student has a library card and has done the laptop training." Jin rewrites it as: "If a student has a library card, then the student may borrow a laptop if and only if the student has done the training." Let C be "the student has a library card", D be "the student has done the training" and L be "the student may borrow a laptop". (4 points)

(a) Write the library's rule and Jin's version in symbols.

(b) Use a truth table, letters in the order C, D, L, to test whether the two versions are logically equivalent. Name every row where they disagree.

(c) Describe in words a student about whom the two versions disagree.

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