Discrete Mathematics and Proof Unit 1 worksheet: A conditional and its contrapositive

Name _____ Date _____ Period _____

1. Write the contrapositive of each statement in symbols. Simplify it so that every $\neg$ sits on a single letter and no letter carries two of them.

(a) $\neg P \to \neg Q$

(b) $Q \to \neg R$

(c) $(P \wedge Q) \to R$

(d) $\neg P \to (Q \wedge \neg R)$

(e) $(P \vee \neg Q) \to (R \wedge S)$

2. Write the contrapositive of each statement as a plain sentence. Do not begin a part with "it is not the case". You do not have to decide whether the statements are true.

(a) If a triangle has two equal angles, then it has two equal sides.

(b) If the train is on time and the bus is not late, then Elena reaches school before 8 a.m.

(c) Every multiple of 6 is even. (First write it as an if-then statement about an integer n.)

(d) If a soccer team scores at least 3 goals in a match, then it wins the match.

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3. Here x and y are real numbers. In (a) and (b), write the contrapositive and use it to decide whether the statement is true. Give a counterexample to any false statement.

(a) If xy is irrational, then x is irrational or y is irrational.

(b) If $x + y$ is irrational, then x is irrational and y is irrational.

(c) Is the converse of the statement in (a) true? Explain.

4. A bus company's log for eight trips records whether it rained during the trip and whether the bus arrived late.

Trip	1	2	3	4	5	6	7	8
Rained	yes	no	no	yes	no	no	yes	no
Late	yes	no	yes	yes	no	yes	yes	no

Consider the claim about these trips: "If it rained during a trip, then the bus arrived late."

(a) Write the converse, the inverse and the contrapositive of the claim.

(b) For each of the four statements, decide whether it holds for every trip in the log, and name every trip that breaks it.

(c) A ninth trip is added to the log, and the claim no longer holds for all nine trips. What happened on the ninth trip? Which of the other three statements does that trip break?

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5. Let x be a real number.

(a) Prove, by proving the contrapositive, that if x is irrational, then $5x - 2$ is irrational.

(b) Show that the statement "if x is irrational, then x^2 is irrational" is false.

Statements	Reasons

6. Malik wants to prove that for every integer n, if $n^2 + 1$ is even, then n is odd. He writes:

> The contrapositive is: if n is odd, then $n^2 + 1$ is even. Suppose n is odd, so $n = 2k + 1$ for an integer k. Then $n^2 + 1 = 4k^2 + 4k + 2 = 2(2k^2 + 2k + 1)$, which is even. So the statement is proved.

(a) Which of the converse, the inverse and the contrapositive of the statement did Malik actually prove?

(b) Does his work prove the statement? Explain.

(c) Write the true contrapositive of the statement, and prove it.

Statements	Reasons

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