Discrete Mathematics and Proof Unit 1 worksheet: De Morgan's laws for statements

Name _____ Date _____ Period _____

1. Write the negation of each sentence as a plain sentence. Do not begin with "it is not the case that".

(a) The museum is open on Monday and admission is free.

(b) The bus was late or the driver took a different route.

(c) The soup is neither salty nor spicy.

(d) The dough has risen, the oven is hot and the tray is greased.

2. Rewrite each statement so that every $\neg$ sits on a single letter, simplifying any double negation. Show each use of a De Morgan law.

(a) $\neg(P \wedge \neg Q \wedge R)$

(b) $\neg\big((P \vee Q) \wedge \neg R\big)$

(c) $\neg\big(\neg(P \wedge Q) \vee R\big)$

(d) $\neg\big((P \wedge \neg Q) \vee (\neg P \wedge R)\big)$

3. The condition $-3 < x \leq 5$ means "$x > -3$ and $x \leq 5$". Four students wrote its negation as follows.

(a) $x < -3$ or $x > 5$

(b) $x \leq -3$ and $x > 5$

(c) $x \leq -3$ or $x \geq 5$

(d) $x \leq -3$ or $x > 5$

Say which are correct. For each one that is wrong, find every real number x at which it disagrees with the true negation.

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4. A website turns down a new account when the username is already taken, or the password is shorter than 8 characters, or the email address is already in use. Let T be "the username is already taken", S be "the password is shorter than 8 characters" and E be "the email address is already in use".

(a) Write in symbols the condition for turning an account down. Then use the De Morgan laws to write the condition for accepting it, with every $\neg$ on a single letter, and say it in words.

(b) The site's programmer coded the accept condition as $\neg T \vee \neg S \vee \neg E$. Describe an account this lets through that the site should turn down, and say which statement the programmer's condition is really the negation of.

5. Use a truth table (letters P, Q, R in the fixed order) to decide whether the statements in each pair are logically equivalent. When they are not, name every row where they disagree.

(a) $\neg\big(P \wedge (Q \vee R)\big)$ and $\neg P \vee (\neg Q \wedge \neg R)$

(b) $\neg\big(P \vee (Q \wedge R)\big)$ and $\neg P \wedge \neg Q \wedge \neg R$

(c) The pair that is not equivalent came from a student's attempt to push the $\neg$ inside. Find the mistake, and write the correct statement with every $\neg$ on a single letter.

6. A statement uses only the connectives $\wedge$ and $\vee$, and every $\neg$ in it sits on a single letter. A classmate proposes a shortcut for writing its negation: swap every $\wedge$ with $\vee$ and every $\vee$ with $\wedge$, put a $\neg$ on every letter that has none, and remove the $\neg$ from every letter that has one.

Let S be $\big(\neg P \wedge (Q \vee \neg R)\big) \vee (R \wedge \neg Q)$.

(a) Use the shortcut to write the negation of S. Then find $\neg S$ again by applying the De Morgan laws one connective at a time, and check that the two agree.

(b) Find the truth value of S and of your answer to (a) when P is true, Q is false and R is true.

(c) Explain why the shortcut works for every statement of this kind.

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