Discrete Mathematics and Proof Unit 1 worksheet: Exclusive or and logic circuits

Name _____ Date _____ Period _____

1. Use the laws of exclusive or to simplify each statement to a single letter, its negation, T or F.

(a) $Q \oplus (\neg P \oplus Q)$

(b) $(P \oplus \mathrm{T}) \oplus (\neg P \oplus \mathrm{T})$

(c) $\neg P \oplus \neg(Q \oplus P) \oplus Q$

(d) $(P \oplus \neg Q) \oplus (\neg Q \oplus R) \oplus (R \oplus P \oplus Q)$

2. Yuna and Andre share a secret key $K = 11000110$. To send an 8-bit message M, Yuna sends $C = M \oplus K$, where $\oplus$ is taken position by position: the first bit of C is the exclusive or of the first bits of M and K, and so on.

(a) Find C when $M = 01101001$.

(b) Andre receives C and works out $C \oplus K$, position by position. Use the laws of exclusive or to explain why this always gives back M.

(c) An eavesdropper learns both M and C from part (a). The next day the same key is used, and the eavesdropper sees $C' = 10011100$. Find the next day's message, and explain how the eavesdropper can.

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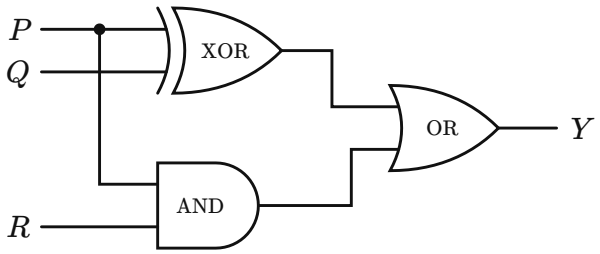
3. A long hallway has one light and three switches, one at each door. Let A, B and C be "switch A is up", "switch B is up" and "switch C is up". The electrician's rules: the light is off when all three switches are down, and flipping any one switch, from any position, changes the light.

(a) Explain why the light L must be $A \oplus B \oplus C$.

(b) Describe a circuit for L made of XOR gates only, and count its gates.

(c) Suppose only AND, OR and NOT gates are available. Write the statement the table method gives for L (rows in the order A, B, C). Count its two-input gates, building each term separately and using one NOT gate for each letter that is negated.

4. The circuit has inputs P, Q, R and output Y. The dot shows where P's wire branches; the branch crosses Q's wire without connecting to it.



(a) Write Y as a statement in P, Q and R.

(b) Make the truth table of Y, rows in the order P, Q, R, and list the rows on which Y is false.

(c) The branch from P is moved so that it carries Q into the AND gate instead. Show that this does not change the output on any row.

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5. An engineer is handed this table for the output Z of a circuit with inputs A, B and C.

Row	A	B	C	Z
1	T	T	T	F
2	T	T	F	T
3	T	F	T	T
4	T	F	F	T
5	F	T	T	T
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

(a) Write the statement the table method gives for Z. Count its two-input gates, building each term separately and using one NOT gate for each letter that is negated.

(b) Find a circuit for Z with only two gates, one of them an XOR gate, and explain why it has the column above.

6. A circuit has two inputs, P and Q, and is made of XOR gates only (as many as you like, wired any way without loops; a wire may branch).

(a) Explain why every wire in the circuit carries a statement equivalent to one of F, P, Q, $P \oplus Q$.

(b) Conclude that no such circuit computes $P \wedge Q$, and none computes $P \vee Q$.

(c) Now a wire that is always T may also be used. How many of the 16 possible columns of a two-letter truth table can such circuits compute? List the statements.

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