Discrete Mathematics and Proof Unit 1 worksheet: Tautologies and contradictions

Name _____ Date _____ Period _____

1. Classify each statement as a tautology, a contradiction or neither. When it is neither, name the rows (letters P, Q in the fixed order) where it is T and the rows where it is F.

(a) $\neg P \to (P \to Q)$

(b) $(P \leftrightarrow Q) \wedge (\neg P \wedge Q)$

(c) $(P \wedge Q) \leftrightarrow (P \vee Q)$

(d) $(P \to Q) \vee (P \to \neg Q)$

(e) $\neg(P \wedge Q) \to \neg P$

(f) For which of the statements in (a) to (e) is the negation a tautology? Answer without building any new table.

2. Decide each part without writing all eight rows of the table (letters P, Q, R in the fixed order).

(a) Show that $\big(P \wedge (Q \vee R)\big) \to \big((P \wedge Q) \vee R\big)$ is a tautology by trying to make it false.

(b) Show that $(P \leftrightarrow Q) \wedge (Q \leftrightarrow R) \wedge (P \wedge \neg R)$ is a contradiction by trying to make it true.

(c) Is $\big(P \to (Q \vee R)\big) \to (P \to Q)$ a tautology? If not, name every row in which it is false.

3. (a) Use a truth table to show that $P \to (Q \to P)$ is a tautology.

(b) Replace P by $R \vee S$ and Q by $\neg R$ at every place they occur. Write the new statement, and explain without a table why it is a tautology.

(c) Isaac replaces only the last P by $R \vee S$ and leaves the rest as it was. Write his statement, and show that it is not a tautology by giving truth values of its letters that make it false.

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4. A and B are statements built from the same letters.

(a) Explain why, if $A \wedge B$ is a tautology, then A and B are each tautologies.

(b) Give an example of statements A and B, built from the one letter P, for which $A \vee B$ is a tautology but neither A nor B is.

(c) Explain why, if $A \vee B$ is a contradiction, then A and B are each contradictions.

5. A forecast says: "Tomorrow will be windy or rainy." Let W be "tomorrow is windy" and R be "tomorrow is rainy". Two listeners each add a premise and draw a conclusion.

Argument 1: "Tomorrow will be windy or rainy. It will not be windy. So it will be rainy."

Argument 2: "Tomorrow will be windy or rainy. It will be windy. So it will not be rainy."

(a) Write each argument as one conditional.

(b) Decide whether each conditional is a tautology, and so whether each argument is valid. For an argument that is not valid, name the counterexample row (letters W, R in the fixed order) and say what kind of day it describes.

6. Two statements are logically equivalent exactly when their biconditional is a tautology. Use this test to decide whether the statements in each pair are logically equivalent. When they are not, name every row (letters in the order P, Q, R) where the biconditional is false.

(a) $\neg(P \leftrightarrow Q)$ and $\neg P \leftrightarrow Q$

(b) $(P \wedge Q) \rightarrow R$ and $(P \rightarrow R) \wedge (Q \rightarrow R)$

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