Discrete Mathematics and Proof Unit 1 worksheet: Truth table for the conditional (answer key)

ANSWERS AT A GLANCE

1. (a) row 1, true (b) row 2, false (c) row 3, true (d) row 4, true (e) only (b) changes; it becomes true
2. (a) P true, Q false, R true (b) P false, Q false, R false (c) P false, Q true, R cannot be determined (d) P true, Q false, R true
3. (a) Yes. $P \to Q$ is false only in row 2 and $Q \to P$ only in row 3, so no row makes both false. (b) rows 1 and 4 (c) $P \to Q$ is true, in row 3; $Q \to P$ is false.
4. (a) A and 9 (b) M has P false, so it is in row 3 or 4, both true. 6 has Q true, so it is in row 1 or 3, both true.
5. (a) $(S \vee W) \to C$ (b) last column T, F, T, F, T, F, T, T (c) A calm day without snow is row 7 or row 8; the notice is true in both, including row 7, where the lift is closed, so a closed lift on such a day does not break it.

WORKED SOLUTIONS

1. Let $n = 45$. For each statement, say whether its hypothesis and its conclusion are true or false, name the row of the table for $P \to Q$ the statement sits in, and give its truth value.

   (a) If n is a multiple of 9, then n is a multiple of 3.

   (b) If n is a multiple of 9, then n is even.

   (c) If n is a perfect square, then n is a multiple of 5.

   (d) If n is prime, then $n < 10$.

   (e) Now let $n = 90$. Which of the statements (a) to (d) change truth value?

   ANSWER

   (a) row 1, true (b) row 2, false (c) row 3, true (d) row 4, true (e) only (b) changes; it becomes true

   SOLUTION

   (a) 45 = 9 × 5, so the hypothesis is true, and 45 = 3 × 15, so the conclusion is true. Row 1: true.

   (b) The hypothesis is true, as in (a), but 45 is odd, so the conclusion is false. Row 2, the only false row: false.

   (c) $6^2 = 36$ and $7^2 = 49$, so 45 is not a perfect square: the hypothesis is false. 45 = 5 × 9, so the conclusion is true. Row 3: true.

   (d) 45 = 5 × 9 is not prime, so the hypothesis is false, and 45 < 10 is false too. Row 4: true, vacuously.

   (e) 90 = 9 × 10 and 90 is even, so (b) moves to row 1 and becomes true. (a) stays in row 1, since 90 = 3 × 30. 90 is neither a perfect square nor prime, so (c) and (d) still have false hypotheses and stay true. Only (b) changes.

2. In each part, the facts listed are all true. Find the truth value of each of P, Q and R, or say that it cannot be determined.

   (a) $P \to Q$ is false, and $Q \vee R$ is true.

   (b) $P \to Q$ is true, $Q \to R$ is true, and R is false.

   (c) $Q \to P$ is false, and $P \to R$ is true.

   (d) $P \to \neg Q$ is true, and $P \wedge R$ is true.

   ANSWER

   (a) P true, Q false, R true (b) P false, Q false, R false (c) P false, Q true, R cannot be determined (d) P true, Q false, R true

   SOLUTION

Free sample from math.wiki for classroom use. Do not post. math.wiki

(a) $P \to Q$ is false only in row 2 of its table, so P is true and Q is false. Then $Q \vee R$ is true with Q false, so R is true.

(b) R is false and $Q \to R$ is true. A conditional with a false conclusion is true only in row 4, where its hypothesis is false too, so Q is false. The same reasoning on $P \to Q$, now with Q false, gives P false.

(c) $Q \to P$ is false, so it is in row 2 of its own table: Q is true and P is false. Then $P \to R$ has a false hypothesis, so it is true whether R is true or false (row 3 or row 4). R cannot be determined.

(d) $P \wedge R$ is true, so P and R are both true. $P \to \neg Q$ is true and its hypothesis P is true, so it is in row 1 and its conclusion $\neg Q$ is true: Q is false.

3. Naomi claims: "Whatever statements P and Q are, at least one of $P \to Q$ and $Q \to P$ is true."

(a) Is she right? Explain with the rows of the two tables.

(b) In which rows of the table, letters P, Q in the fixed order, are $P \to Q$ and $Q \to P$ both true?

(c) Let P be "7 is even" and Q be "7 is prime". Which of the two conditionals is true, and which row is it in?

ANSWER

(a) Yes. $P \to Q$ is false only in row 2 and $Q \to P$ only in row 3, so no row makes both false. (b) rows 1 and 4 (c) $P \to Q$ is true, in row 3; $Q \to P$ is false.

SOLUTION

(a) $P \to Q$ is false only where P is true and Q is false, which is row 2. $Q \to P$ is false only where Q is true and P is false, which is row 3. For both to be false at once, one row would have to be row 2 and row 3 together, which is impossible. So in every row at least one of them is true, and Naomi is right.

(b) Both are true in the rows that are neither row 2 nor row 3: rows 1 and 4, where P and Q have the same truth value.

(c) 7 is odd, so P is false; 7 is prime, so Q is true. That is row 3: $P \to Q$ has a false hypothesis and is true, while $Q \to P$ has a true hypothesis and a false conclusion, so it is false.

4. Each card in a pack has a letter on one side and a whole number on the other. A claim about the pack says: "If a card has a vowel on one side, then it has an even number on the other side." Four cards lie on a table, showing A, M, 6 and 9.

(a) Which cards must you turn over to find out whether any of these four breaks the claim? Turn over no card you do not need to.

(b) Let P be "the card has a vowel" and Q be "the card has an even number". For each card you leave face down, use the rows of the table for $P \to Q$ to explain why it cannot break the claim.

ANSWER

(a) A and 9 (b) M has P false, so it is in row 3 or 4, both true. 6 has Q true, so it is in row 1 or 3, both true.

SOLUTION

(a) A card breaks the claim only if it sits in row 2: a vowel together with an odd number. A shows a vowel, so P is true, and the number on its back could be odd: turn it over. 9 shows an odd number, so Q is false, and if the letter on its back is a vowel the card is in row 2: turn it over.

(b) M shows a consonant, so P is false and the card is in row 3 or 4 whatever its number is; both rows are true, because the claim says nothing about consonants. 6 shows an even number, so Q is true and the card is in row 1 or 3 whatever its letter is; both rows are true. Turning over the 6 is the usual slip: it tests "if even, then vowel", the converse, which the claim never made.

Free sample from math.wiki for classroom use. Do not post. math.wiki

5. A ski resort posts the notice: "If it snows or the wind is strong, the upper lift is closed." Let S be "it snows", W be "the wind is strong" and C be "the upper lift is closed".

(a) Write the notice as a statement in S, W and C.

(b) Build its truth table, rows in the order S, W, C, with a helper column for $S \vee W$.

(c) Sofia, who runs the lift, reads the notice as also promising that the lift is open on every calm day without snow. Use the table to explain why the notice makes no such promise.

ANSWER

(a) $(S \vee W) \to C$ (b) last column T, F, T, F, T, F, T, T (c) A calm day without snow is row 7 or row 8; the notice is true in both, including row 7, where the lift is closed, so a closed lift on such a day does not break it.

SOLUTION

(a) The hypothesis is the or, the conclusion is C: $(S \vee W) \to C$.

(b)

Row	S	W	C	$S \vee W$	$(S \vee W) \to C$
1	T	T	T	T	T
2	T	T	F	T	F
3	T	F	T	T	T
4	T	F	F	T	F
5	F	T	T	T	T
6	F	T	F	T	F
7	F	F	T	F	T
8	F	F	F	F	T

The notice is broken exactly in rows 2, 4 and 6: it snows or the wind is strong, and the lift is open anyway.

(c) A calm day without snow has S and W both false, which is row 7 or row 8. There $S \vee W$ is false, so the notice is true vacuously in both rows, whatever the lift does. In particular row 7, a calm, snowless day with the lift closed, does not break it. The notice only says what happens in bad weather; Sofia is reading in the promise "if neither, then open", which it never made.

Free sample from math.wiki for classroom use. Do not post. math.wiki