Deductive reasoning

How do you show that a claim holds for every figure of a kind, and not just for the few you have drawn and measured?

WORDS TO KNOW

Deductive reasoning: reaching a conclusion that must be true whenever the statements it starts from are true, one justified step at a time.

Given: what the problem tells you about this figure, such as $l \parallel m$.

Theorem: a fact already proved by deduction, such as the Vertical Angles Theorem.

Inductive reasoning: noticing a pattern in examples and guessing it always holds. It gives a conjecture, not a proof.

Postulate: a basic fact accepted without proof, such as the segment addition postulate.

$\therefore$**:** "therefore", written before the conclusion.

KEY IDEAS

1. Every step needs a reason

A deduction is a chain of statements. Each one is justified by something already accepted, or by an earlier line.

> The four kinds of reason in geometry:
>
> **givens**, **definitions**, **postulates** and **theorems** (facts already proved).

Because the chain uses only the givens and facts true for every such figure, the conclusion holds for **every** figure that fits the givens, not just the one on the page. Once proved, it becomes a theorem and can be a reason in later proofs.

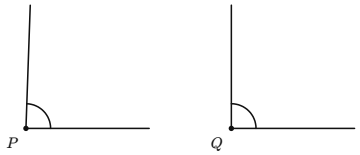
2. Two laws that make the steps valid

> **Law of detachment.** If "if p, then q" is accepted and p is true, then q is true.
>
> **Law of syllogism.** If "if p, then q" and "if q, then r" are both accepted, then "if p, then r".

Applying a theorem to your figure is the law of detachment: the theorem is the if-then, and an earlier line supplies its if-part.

3. A drawing is never a reason

One of these angles is 90° and the other is 88°. You cannot tell which by looking. In fact $\angle P$ is the 88° one. A drawing may be read for position, such as which points lie on a line and in what order. Equal lengths, equal angles and parallel lines must be given or deduced.



	Inductive	Deductive
Starts from	examples, drawings, measurements	givens and accepted facts
Gives	a conjecture: probable	a conclusion: certain, if the start is true
Covers	the cases checked	every figure that fits the givens

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WATCH OUT

- **Do not run a theorem backwards.** "If two angles form a linear pair, they are supplementary" does not say supplementary angles form a linear pair. Having the then-part does not give you the if-part.
- **The whole if-part must be met.** A theorem about parallel lines says nothing unless $l \parallel m$ is given or proved.
- **Deduction cannot fix a false start.** A valid chain from a false given can end anywhere. The conclusion is only as sure as what it started from.
- **No circles.** You cannot use fact A to prove fact B and then B to prove A.

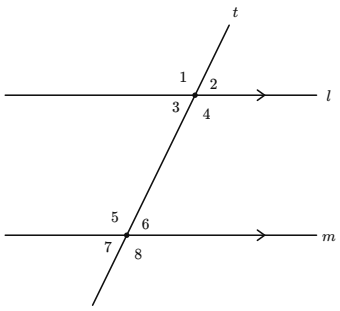
WORKED EXAMPLES

Example 1. Deductive or inductive? (a) Six triangles are measured and each angle sum is 180°, so every triangle's angles add to 180°. (b) $\angle 1$ and $\angle 2$ form a linear pair, and linear pairs are supplementary, so $m\angle 1 + m\angle 2 = 180°$. (c) Lines l and m look parallel in the diagram, so their alternate interior angles are equal.

1. (a) Six examples support the claim, but a seventh triangle has not been checked. Inductive: a conjecture.
2. (b) The Linear Pair Theorem is an if-then, and the first sentence gives its if-part. Deductive, by the law of detachment.
3. (c) The step would be valid, but $l \parallel m$ was read off the drawing, not given. Not a sound deduction.

Answer: (a) inductive, (b) deductive, (c) neither: the starting fact was never given.

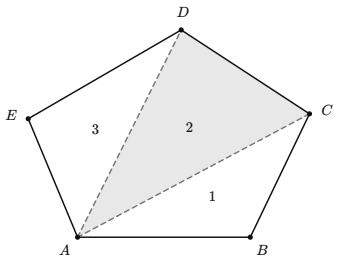
Example 2. Lines $l \parallel m$ are cut by t, and $m\angle 2 = 64°$. Find $m\angle 6$, $m\angle 5$ and $m\angle 3$ with a reason for each.



1. $\angle 2$ and $\angle 6$ are in the same position at the two crossings, so they are corresponding angles. The lines are parallel, so by the Corresponding Angles Theorem, $m\angle 6 = 64°$.
2. $\angle 5$ and $\angle 6$ form a linear pair. By the Linear Pair Theorem, $m\angle 5 = 180° - 64° = 116°$.
3. $\angle 3$ is opposite $\angle 2$ at the same crossing. By the Vertical Angles Theorem, $m\angle 3 = 64°$.

Answer: $m\angle 6 = 64°$, $m\angle 5 = 116°$, $m\angle 3 = 64°$. **Check:** 64 + 116 = 180.

Example 3. Prove that the angles of a convex polygon with n sides add to $180(n-2)$ degrees. Use it for a hexagon.



Plan: cut the polygon into triangles and use the Triangle Angle Sum Theorem.

1. Pick one corner A. Draw a diagonal from A to each of the $n-3$ corners that are not A or its neighbors.
2. The polygon is convex, so each diagonal runs inside it. The diagonals cut it into $n-2$ triangles.
3. Each triangle's angles add to 180° (Triangle Angle Sum Theorem).
4. Together, the triangles' angles make up exactly the polygon's angles (angle addition postulate at each corner). So the sum is $180(n-2)$ degrees.

Answer: for a hexagon, 180(6 – 2) = 180 × 4 = 720°.

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