Midpoints and perpendicular bisectors on the grid

Two points are given by their coordinates. Find the point halfway between them, run that backwards to find a missing end, and write the line of every point equally far from both.

WORDS TO KNOW

Midpoint: the point on a segment that is equally far from both ends.

Perpendicular bisector: the line through the midpoint of a segment that meets it at a right angle. Every point on it is equally far from the two ends.

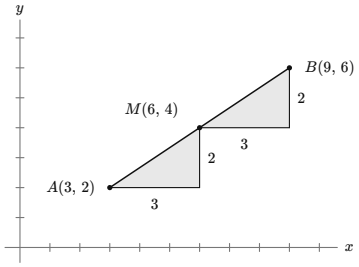
(x_1, y_1) and (x_2, y_2): the coordinates of the first and second end. Which one you call first does not matter.

Negative reciprocal: flip a slope and change its sign. The negative reciprocal of $\frac{2}{3}$ is $-\frac{3}{2}$. Perpendicular slopes multiply to −1.

KEY IDEAS

1. The midpoint is the average of the coordinates

Walk from one end to the other and stop halfway: make half the move across and half the move up. For (3, 2) to (9, 6), the move is 6 right and 4 up, so half is 3 right and 2 up, landing on (6, 4).



$$M = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2} \right)$$

Average the x-coordinates, then average the y-coordinates.

2. Run it backwards to find a missing end

The move from M to the far end is the same as the move from the near end to M. So make that move once more, starting at M.

If $M = (x_M, y_M)$ is the midpoint and (x_1, y_1) is one end, the other end is

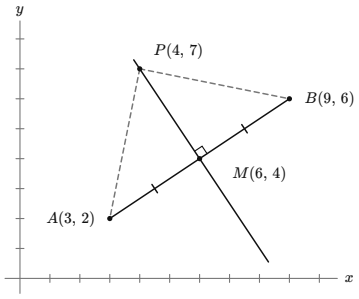
$$(x_2,\ y_2) = (2x_M - x_1,\ 2y_M - y_1)$$

3. The perpendicular bisector: a point and a slope

The line needs a point and a slope, and the two words in its name give one each.

1. **Point:** the midpoint M.
2. **Slope:** the negative reciprocal of the segment's slope.
3. **Equation:** point-slope form, $y - y_M = m(x - x_M)$, then solve for y.

Level segment: the bisector is the upright line $x = x_M$. Upright segment: it is the level line $y = y_M$.



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WATCH OUT

- **Running backwards means doubling, not averaging.** The missing end lies beyond M, as far past it as A is before it. Double M and subtract A.
- **Do not flip without changing the sign** (or change the sign without flipping). The new slope times the old one must be −1.
- **A level or upright segment has no negative reciprocal to take.** Slope 0 has no reciprocal, and an upright segment has no slope. Use $x = x_M$ or $y = y_M$.
- **A midpoint can have fractions** even when the ends are whole numbers, such as (−7, −1) and (4, −6), whose midpoint is $\left(-\frac{3}{2}, -\frac{7}{2}\right)$. Nothing has gone wrong.

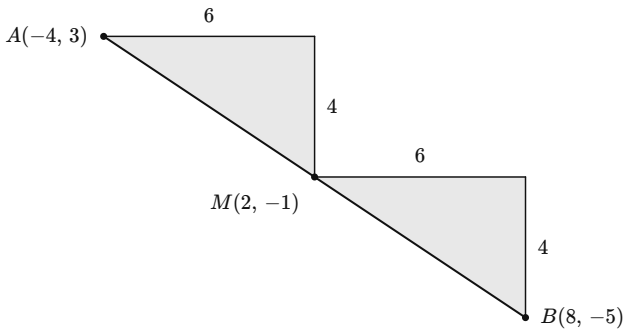
WORKED EXAMPLES

Example 1. Find the midpoint of the segment from (−3, 8) to (5, 2).

1. Average the x-coordinates: $\frac{-3+5}{2} = \frac{2}{2} = 1$.
2. Average the y-coordinates: $\frac{8+2}{2} = \frac{10}{2} = 5$.

Answer: (1, 5). **Check:** from (−3, 8) to (1, 5) is 4 right and 3 down, and from (1, 5) to (5, 2) is also 4 right and 3 down.

Example 2. $M = (2, -1)$ **is the midpoint of** $\overline{AB}$**, and** $A = (-4, 3)$**. Find** B**.**



1. The move from A to M is 2 − (−4) = 6 right and −1 − 3 = −4, so 4 down.
2. Make the same move again from M: (2 + 6, −1 − 4) = (8, −5).
3. The formula gives the same: (2 · 2 − (−4), 2 · (−1) − 3) = (8, −5).

Answer: $B = (8, -5)$. **Check:** $\left(\frac{-4+8}{2}, \frac{3+(-5)}{2}\right) = (2, -1)$, which is M.

Example 3. Find the equation of the perpendicular bisector of the segment from $A = (-6, 1)$ **to** $B = (2, 5)$**.**

1. Midpoint: $\left(\frac{-6+2}{2}, \frac{1+5}{2}\right) = (-2, 3)$.
2. Slope of $\overline{AB}$: $\frac{5-1}{2-(-6)} = \frac{4}{8} = \frac{1}{2}$. The perpendicular slope is −2, since $\frac{1}{2} \cdot (-2) = -1$.
3. Point-slope form through (−2, 3): $y - 3 = -2(x + 2)$, so $y - 3 = -2x - 4$ and $y = -2x - 1$.

Answer: $y = -2x - 1$. **Check:** $P = (0, -1)$ is on the line, and it is equally far from A and B:

$$PA^2 = 6^2 + (-2)^2 = 40 \qquad PB^2 = (-2)^2 + (-6)^2 = 40$$

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