Statements and negation

A statement is true or false, and never both. What sentence says exactly the opposite, and how do you record what "not" does in a truth table?

WORDS TO KNOW

Statement: a sentence that is either true or false, and not both. Nobody has to know which yet.

Truth value: T (true) or F (false).

Free variable: a letter like x that has no value yet. A sentence with one, such as $x + 4 = 10$, is not a statement until x is fixed.

Negation: the statement that is true exactly when the original is false. Written $\neg P$, read "not P".

Truth table: a table with one row for each way the letters can be true or false, and a column for each statement built from them.

KEY IDEAS

1. What counts as a statement

A statement makes a claim that settles one way or the other. Questions, instructions and opinions do not.



Sentence	Statement?
"54 is divisible by 9."	yes, true
"Every even number greater than 2 is the sum of two primes."	yes, though nobody has proved it either way
"Close the door." / "Is 17 prime?"	no: an instruction, a question
"$x + 4 = 10$"	not until x has a value

2. Negation swaps true and false

To negate, work "not" into the sentence, or write "It is not the case that ...". The negation must be true in **exactly** the cases where the original is false.

For every statement P, exactly one of P and $\neg P$ is true.

Double negation: $\neg(\neg P)$ has the same truth value as P. An even number of ¬ signs gives back P; an odd number gives $\neg P$.

3. The truth table for not

Row	P	$\neg P$
1	T	F
2	F	T

To build the column for $\neg X$, swap T and F in the column for X, row by row. Nothing inside X is recomputed.

A ¬ covers only the letter or group in parentheses right after it: $\neg P \wedge Q$ means $(\neg P) \wedge Q$, which is different from $\neg(P \wedge Q)$.

Adding a ¬ never adds rows. With n letters there are 2^n rows.

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WATCH OUT

- **Include the boundary when you negate a comparison.** The negation of $t < 0$ is $t \geq 0$, not $t > 0$. At $t = 0$ the original is false, so the negation must be true there.
- **The table does not tell you which row you are in.** It says what $\neg P$ is for each value of P. Whether a particular P, such as "51 is prime", is true is a separate check: 51 = 3 × 17, so it is false and sits in row 2.
- **Not knowing the truth value is not the same as having none.** An unproved claim is still a statement.
- **Negation flips a claim; it does not weaken it.** $\neg P$ does not mean "P is unlikely". It is a full statement, true in exactly the opposite cases.

WORKED EXAMPLES

Example 1. Which are statements? Give the truth value of each statement. (a) "36 is a perfect square." (b) "What time does the shop close?" (c) "$y - 3 = 11$." (d) "Pineapple belongs on pizza."

1. (a) It is true or false, and in fact true, since $36 = 6^2$.
2. (b) A question asks; it does not claim anything.
3. (c) y has no value. It is true for $y = 14$ and false for other values, so on its own it is neither.
4. (d) A preference. No fact settles it.

Answer: only (a) is a statement, and it is true.

Example 2. n is one particular whole number. Write the negation of "$n > 12$". Why is it not "$n < 12$"?

1. The negation must be true exactly when "$n > 12$" is false.
2. "$n > 12$" is false when n is 12 or smaller. At $n = 12$: 12 > 12 is false.
3. So $n = 12$ must make the negation true. "$n < 12$" is false at $n = 12$, so it misses that case.

Answer: the negation is "$n \leq 12$".

Example 3. The column for $P \vee Q$ ("P or Q") in rows TT, TF, FT, FF is T, T, T, F. Build the column for $\neg(P \vee Q)$ and say when it is true.

1. The $\neg$ is in front of parentheses, so it negates the whole of $P \vee Q$.
2. Swap each entry of the $P \vee Q$ column. The P and Q columns are not used again.

Row	P	Q	$P \vee Q$	$\neg(P \vee Q)$
1	T	T	T	F
2	T	F	T	F
3	F	T	T	F
4	F	F	F	T

Answer: F, F, F, T. It is true only in row 4, where P and Q are both false: it says "neither P nor Q". The table still has four rows.

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