Geometry Unit 1 quiz 2: The midpoint formula, Conjecture, and why a pattern is not a proof, Counterexample

Name _____ Date _____ Period _____

8 questions, 21 points

1. M is the midpoint of $\overline{AB}$, and N is the midpoint of $\overline{AM}$. If $A = (-4, 3)$ and $N = (-7, 2)$, find M and B. (2 points)

2. $A = (6, -2)$, and B is a point on the x-axis. The midpoint of $\overline{AB}$ lies on the line $y = x$. Find B and the midpoint. (2 points)

3. Claim: for every segment $\overline{AB}$ in the coordinate plane, the midpoint M of $\overline{AB}$ is closer to the origin O than A is, and closer to O than B is. Find a counterexample, and show exactly that it fails. (2 points)

4. $M = (1, 2)$ is the midpoint of $\overline{AB}$, where $A = (-3, k)$, and $AB = 10$. Find every possible value of k, and the point B that goes with each. (3 points)

5. (3 points)

(a) Write "no triangle has two obtuse angles" as a for-every statement, naming the collection, and say what a single triangle could do to it.

(b) Write "some rectangle has a diagonal exactly twice as long as its width" as a there-is statement, and settle it using a rectangle of width 1 and length $\sqrt{3}$.

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6. A proof of a statement R about triangles is correct in every step, except that one step uses conjecture C, which is still open. (3 points)

(a) Is R a theorem? Explain.

(b) Later, C is proved. What is the status of R now?

(c) Suppose instead that C is disproved. Does that make R false? Explain.

7. The angle sums of a triangle, a quadrilateral and a pentagon are $180°$, $360°$ and $540°$. A student predicts $720°$ for a hexagon, following the rule $f(n) = 180(n-2)$ for a polygon with n sides. (3 points)

(a) Find a number c so that the rule $g(n) = 180(n-2) + c(n-3)(n-4)(n-5)$ also gives 180, 360, 540 at $n = 3, 4, 5$ but gives 900 at $n = 6$.

(b) The three given sums do not decide between f and g. What does? Settle the angle sum of a convex hexagon, taking the angle sum theorem for triangles as proved.

8. Claim: for every real number m other than 0, the line $y = mx + 1$ intersects the line $y = 2x - 3$. For each value of m, decide whether it is a counterexample, and say why.

(a) $m = -1$ (b) $m = 0$ (c) $m = 2$

(3 points)

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