Geometry Unit 1 test: Points, lines, planes and the language of proof

Name _____ Date _____ Period _____

12 questions, 49 points

1. Two lines are **perpendicular** if they meet to form a right angle. Both statements below are about two different lines in a plane, and both are true. (2 points)

(a) If two lines are perpendicular, then they intersect.

(b) If two lines are perpendicular, then all four angles they form are right angles.

Write the converse of each statement. Say whether the converse is true, and back up your answer: name the reason that proves it, or give a counterexample.

2. The points are $A(-3,-2)$, $B(1,1)$ and $C(9,7)$. (3 points)

(a) Use the distance formula to find AB, BC and AC.

(b) Use the distance test to decide whether B lies on $\overline{AC}$.

(c) The point $D(5,5)$ is also given. Without the distance formula, decide whether D lies on the line through A and C. Then say whether $AD + DC$ equals AC or is greater than it, and why.

3. Triangle PQR has $P=(-4,3)$, $Q=(2,7)$ and $R=(6,-1)$. M is the midpoint of $\overline{PQ}$ and N is the midpoint of $\overline{PR}$. (3 points)

(a) Find M and N.

(b) Show that $\overline{MN}$ is parallel to $\overline{QR}$ and half as long.

4. Let $A=(-1,2)$ and $B=(5,2)$. For a point P, consider the statement

"P is the midpoint of $\overline{AB}$ if and only if $PA=3$." (3 points)

(a) Show that the forward direction holds.

(b) Show that the backward direction fails, with a point P that has whole-number coordinates.

(c) Replace "$PA=3$" by "$PA=3$ and $PB=3$". Show that the new statement is true for every point P.

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5. Claim: for all points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance $PQ = |x_2 - x_1| + |y_2 - y_1|$. (4 points)

(a) Give a counterexample, and work out both sides exactly.

(b) Find every pair of points P, Q for which the equation does hold, and show why.

6. Two non-vertical lines in the coordinate plane have slopes m_1 and m_2. Statement S: "If $m_1 m_2 = -1$, then the two lines intersect." (4 points)

(a) Explain why S is true.

(b) Write the converse and the inverse of S, and show that both are false with one pair of lines.

(c) The lines $y = ax + 2$ and $y = -\frac{1}{4}x + 5$ do not intersect. Write what the contrapositive of S tells you about a. Then find a exactly.

7. The points are $A(-1, -3)$, $B(5, 5)$, $C(-2, 4)$ and $D(6, -2)$. (5 points)

(a) Write equations of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, and find every point the two lines share. Explain why there is no other.

(b) Show that this point is the midpoint of $\overline{AB}$ and of $\overline{CD}$.

(c) Is $\overleftrightarrow{CD}$ the perpendicular bisector of $\overline{AB}$? Explain.

8. Conjecture: in every triangle, the midpoint of the longest side is the same distance from all three corners. (5 points)

(a) Test the conjecture on the triangle with $A(0, 0)$, $B(6, 0)$ and $C(0, 8)$.

(b) Test it on the triangle with $D(0, 0)$, $E(8, 0)$ and $F(2, 4)$, and settle the conjecture.

(c) Triangle ABC has a right angle and triangle DEF does not. Use this to repair the conjecture, and say what the status of the repaired claim is.

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9. Triangle ABC has $A = (0, 0)$, $B = (6, 0)$ and $C = (3, 4)$. Let P be "triangle ABC is equilateral" and Q be "$AC = BC$". (5 points)

(a) Find AB, AC and BC, and give the truth values of P and Q.

(b) Write $\neg P$ in words.

(c) Build the truth table for $\neg P \wedge \neg Q$, say which row this P and Q fall in, and give the truth value of $\neg P \wedge \neg Q$ for this triangle.

10. A triangle is **isosceles** if at least two of its sides are congruent. (5 points)

(a) Use the distance formula to decide whether the triangle with corners $A(1, 1)$, $B(5, 4)$ and $C(8, 0)$ is isosceles.

(b) Let t be any number other than 0. Show that the triangle with corners $D(-3, 0)$, $E(3, 0)$ and $F(0, t)$ is isosceles.

(c) Find the positive value of t for which all three sides of $\triangle DEF$ are congruent. For that value, is $\triangle DEF$ isosceles under the definition above? Would it be isosceles under the other definition some books use, "exactly two of its sides are congruent"?

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11. Rectangle $ABCD$ has $A(0,0)$, $B(8,0)$, $C(8,6)$ and $D(0,6)$. For a point P, compare $PA^2 + PC^2$ with $PB^2 + PD^2$. (5 points)

(a) Find both sums for $P(2,1)$ and for $P(5,4)$.

(b) State a conjecture about every rectangle $ABCD$ and every point P inside it.

(c) Prove your conjecture. Place the rectangle as $A(0,0)$, $B(w,0)$, $C(w,h)$, $D(0,h)$, and let $P = (x,y)$.

(d) Does the same equation hold in every parallelogram? Test the parallelogram $A(0,0)$, $B(4,0)$, $C(6,2)$, $D(2,2)$, with P the point where its diagonals cross. Then say which hypothesis your proof in (c) used.

12. Points A, B, C and D lie on a line in that order, and $AB = CD$. M is the midpoint of $\overline{BC}$. (5 points)

(a) Take $A(-3,1)$, $B(1,3)$, $C(5,5)$ and $D(9,7)$. Check that $AB = CD$, find M, and check that M is also the midpoint of $\overline{AD}$.

(b) Prove that M is the midpoint of $\overline{AD}$ for every four points that fit the first sentence. Write a two-column proof.

(c) Explain why your work in (a) alone does not prove (b).

Statements	Reasons

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