Geometry Unit 1 worksheet: Biconditional

Name _____ Date _____ Period _____

1. Each statement has the form $P \leftrightarrow Q$, with P the part before "if and only if". Decide P and Q, name the row of the table it falls on, and give its truth value.

(a) The point (3, 5) lies on the line $y = 2x - 1$ if and only if the slope from (0, 0) to (1, 2) is 2.

(b) The segment from (0, 0) to (6, 8) has length 10 if and only if its midpoint is (4, 3).

(c) An angle of 95° is acute if and only if 95 + 85 = 180.

(d) The points (0, 0), (1, 2) and (3, 5) are collinear if and only if a triangle can have two obtuse angles.

(e) The segment from (−2, 1) to (4, 9) has length 14 if and only if its midpoint is (1, 5).

(f) The point (2, 7) lies on the line $y = 3x + 1$ if and only if it lies on the line $y = -x + 9$.

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2. A, B and M are points in a plane. Here are two conditionals.

(A) If M is the midpoint of $\overline{AB}$, then $AM = MB$.

(B) If $AM = MB$, then M is the midpoint of $\overline{AB}$.

(a) Each sentence below says the same thing as (A) or as (B). Say which.

(i) M is the midpoint of $\overline{AB}$ only if $AM = MB$. (ii) $AM = MB$ is sufficient for M to be the midpoint of $\overline{AB}$. (iii) $AM = MB$ if M is the midpoint of $\overline{AB}$. (iv) Being the midpoint of $\overline{AB}$ is necessary for M to have $AM = MB$. (v) $AM = MB$ only if M is the midpoint of $\overline{AB}$. (vi) $AM = MB$ is necessary for M to be the midpoint of $\overline{AB}$.

(b) Decide whether (A) and (B) are true. For one that is false, take $A = (0, 0)$ and $B = (4, 0)$ and give a point M that shows it.

(c) Is "M is the midpoint of $\overline{AB}$ if and only if $AM = MB$" true for all points?

3. For an integer k from −3 to 3, let S be the statement

"The point (k, k^2) lies above the line $y = 2$ if and only if $k > 1$."

(a) For which k is S false? Name the row of the table each one falls on.

(b) Replace "$k > 1$" by a different condition on k so that the new statement is true for every integer k.

4. Fill in the column of each statement. Decide which ones are logically equivalent to $P \leftrightarrow Q$. For each one that is not, name a row where it differs from $P \leftrightarrow Q$.

(a) $(P \vee Q) \rightarrow (P \wedge Q)$ (b) $(P \rightarrow Q) \vee (Q \rightarrow P)$ (c) $(P \wedge Q) \vee \neg(P \vee Q)$ (d) $P \leftrightarrow (P \wedge Q)$

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5. Let $A = (0, 0)$ and $B = (6, 0)$, and let $P = (x, y)$ be any point of the plane. For each condition, decide whether it is sufficient but not necessary, necessary but not sufficient, necessary and sufficient, or neither, for $PA = PB$. Give a point that shows each "not".

(a) $P = (3, 5)$ (b) $x = 3$ (c) P is the midpoint of $\overline{AB}$ (d) $y = 0$ (e) $x > 0$

6. Let $A = (0, 0)$ and $B = (6, 2)$. For a real number k, let $P = (k, 4)$. Show that P is closer to A than to B if and only if $k < 2$. Write your work as a chain of $\iff$ steps, and say why each step can be undone.

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