Geometry Unit 1 worksheet: Conjecture, and why a pattern is not a proof

Name _____ Date _____ Period _____

1. Conjecture C is a claim about every whole number, and it is open. Each line below is a piece of news about C. Say whether the news changes the status of C, and if it does, say what C becomes.

(a) A computer checks C for ten billion more whole numbers, and every one of them passes.

(b) An argument is published, and checked by experts, that shows C holds for every whole number.

(c) One whole number is found for which C fails, and the failure is checked by hand.

(d) A famous mathematician says in an interview that she is sure C is true.

(e) A magazine starts calling it "the C theorem".

2. A row of n squares is built from toothpicks. Each side of a square is one toothpick, and two neighboring squares share the toothpick between them.

(a) Count the toothpicks for rows of 1, 2, 3 and 4 squares, and state a conjecture for a row of n squares.

(b) Give a reason that covers every n, with n kept as a letter.

(c) A row is built from exactly 100 toothpicks. How many squares does it have?

3. O is the origin (0, 0). For each whole number $n \geq 1$, P_n is the point $(n, n+7)$ on the line $y = x + 7$.

(a) Claim A: for every whole number n from 1 to 4, the length OP_n is not a whole number. Settle Claim A.

(b) Claim B: for every whole number $n \geq 1$, the length OP_n is not a whole number. What do your checks in (a) show about Claim B?

(c) Settle Claim B.

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4. Conjecture A: for every whole number $n \geq 1$, the number $n^2 + n + 1$ is prime.

Conjecture B: for every whole number $n \geq 1$, the number $n^2 + n + 1$ is odd.

(a) Explain why A implies B. (Note that $n^2 + n + 1 \geq 3$ for every $n \geq 1$.)

(b) Test A for $n = 1, 2, 3, 4$ and settle A.

(c) After (b), what do you know about B? Explain.

(d) Settle B.

5. Each angle of a regular polygon with n sides measures $\dfrac{180(n-2)}{n}$ degrees. Conjecture: for every whole number $n \geq 3$, each angle of a regular polygon with n sides is a whole number of degrees.

(a) Find the angle for $n = 3, 4, 5, 6$.

(b) Settle the conjecture.

(c) Which values of n from 8 to 12 give a whole number of degrees?

(d) Repair the conjecture by narrowing the collection, and prove the repaired claim. (Hint: $\dfrac{180(n-2)}{n} = 180 - \dfrac{360}{n}$.)

6. Claim: for every whole number n, the last digit of n^5 is the same as the last digit of n.

(a) Check the claim for $n = 0, 1, 2, \ldots, 9$.

(b) Explain why these ten checks prove the claim for every whole number n, even though the claim has infinitely many cases.

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7. Each claim is followed by one fact. Say whether the claim is a for-every claim or a there-is claim, and whether the fact proves the claim, disproves it, or settles nothing.

(a) Claim: every triangle with two sides of equal length has two angles of equal measure. Fact: a triangle with sides 5, 5 and 8 is measured, and two of its angles are equal.

(b) Claim: there is a rectangle whose area and perimeter are the same number. Fact: a square with sides of length 4.

(c) Claim: in every triangle with a right angle, the two sides that form the right angle have different lengths. Fact: a triangle with angles 45°, 45° and 90° has two equal sides forming its right angle.

(d) Claim: there is a triangle with two right angles. Fact: a computer draws a million triangles at random, and none of them has two right angles. After answering, settle this claim using the angle sum theorem.

8. Conjecture: every quadrilateral with four sides of equal length has two diagonals of equal length.

(a) Show that the quadrilateral $PQRS$ with $P(0, 3)$, $Q(4, 0)$, $R(0, -3)$ and $S(-4, 0)$ is a counterexample.

(b) One proposed repair: "every quadrilateral with four sides of equal length, other than $PQRS$, has diagonals of equal length." Show that this repair also fails, using the quadrilateral with corners $(0, 5)$, $(12, 0)$, $(0, -5)$ and $(-12, 0)$.

(c) A second proposed repair: "every quadrilateral with four sides of equal length and at least one right angle has diagonals of equal length." Does $PQRS$ still break this claim? Is the repaired claim a theorem now? Explain.

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