Geometry Unit 1 worksheet: Counterexample

Name _____ Date _____ Period _____

1. Claim: for every point $P(x, y)$ in the coordinate plane, if $x + y = 4$, then the distance from P to the origin O is at most 4. For each point, decide whether it is a counterexample to the claim, and say why.

(a) $P(2, 2)$

(b) $P(5, -1)$

(c) $P(3, 3)$

(d) $P(4, 0)$

2. Each claim is false. Give a counterexample, and show exactly that it meets every hypothesis and makes the conclusion false.

(a) For every real x, $(x + 2)^2 = x^2 + 4$.

(b) For every real x, $\sqrt{x^2} = x$.

(c) For all positive integers a and b, if $a + b$ is even, then a and b are both even.

(d) For every nonzero real x, $x + \dfrac{1}{x} \geq 2$.

(e) For every positive integer n, $3^n > n^3$.

(f) For all real a, b and c, if $ac < bc$, then $a < b$.

3. Claim: for all points A, B and C in the coordinate plane, if $AB = 3$ and $BC = 4$, then $AC = 7$.

(a) Is $A(0, 0)$, $B(3, 0)$, $C(7, 0)$ a counterexample? Explain.

(b) Find a counterexample in which A, B and C are not on one line, and check it with the distance formula.

(c) Find a counterexample in which A, B and C are all on one line, and check it.

4. Each claim is false. Give a counterexample to each, and show why it breaks the claim.

(a) If two angles are supplementary, then one of them is obtuse.

(b) If two segments have the same midpoint, then they have the same length.

(c) If two different lines in space do not intersect, then they are parallel.

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5. Claim: for any three different lines ℓ, m and n, if $\ell \perp m$ and $m \perp n$, then $\ell \parallel n$.

A box has bottom face $ABCD$ and top face $EFGH$, with E above A, F above B, G above C and H above D.

(a) Use lines through edges of the box to give a counterexample, and say why it breaks the claim.

(b) Your counterexample uses lines that do not all lie in one plane. Does it show that the claim is false for three lines that lie in one plane? Explain.

6. Claim: for every real number x, if $x^2 = 5x$, then $x = 5$.

Owen's proof: "Start from $x^2 = 5x$. Divide both sides by x. This gives $x = 5$."

(a) Which step of Owen's proof needs something extra about x? What does it need?

(b) Use your answer to (a) to find a counterexample to the claim, and check it.

(c) Change the hypothesis of the claim so that the claim becomes true.

7. Claim: for all statements p and q, the statement $\neg(p \wedge q)$ is logically equivalent to $\neg p \wedge \neg q$. Find a row of the truth table that is a counterexample, and show the truth value of each statement on that row.

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