Geometry Unit 1 worksheet: Deductive reasoning

Name _____ Date _____ Period _____

1. Each part gives statements to accept as true. Write the conclusion that follows and name the law used (law of detachment or law of syllogism). If no conclusion follows, say so and explain why.

(a) If a point is the midpoint of a segment, then it divides the segment into two segments of equal length. K is the midpoint of $\overline{GH}$.

(b) If an angle measures 30°, then it is acute. If an angle is acute, then its supplement is obtuse.

(c) If a triangle is equilateral, then it is isosceles. $\triangle RST$ is isosceles.

(d) If a polygon is a regular hexagon, then each of its angles measures 120°. Polygon W is not a regular hexagon.

(e) If a quadrilateral is a rectangle, then its diagonals are congruent. If a quadrilateral is a kite, then its diagonals are perpendicular.

2. Points A, O and D lie on a line, with O between A and D. Rays $\overrightarrow{OB}$ and $\overrightarrow{OC}$ lie on the same side of the line, so that the rays come in the order $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$. $m\angle AOB = (4x + 24)^\circ$, $m\angle BOC = (3x)^\circ$ and $m\angle COD = (3x - 4)^\circ$. Find x and the three angle measures, giving a reason for each step.

Free sample from math.wiki for classroom use. Do not post. math.wiki

3. For each argument, say (i) whether every step follows from the statements before it, and (ii) whether the conclusion is certain to be true. Explain each answer.

(a) Every angle that measures less than 100° is acute. $m\angle K = 95^\circ$. So $\angle K$ is acute.

(b) Every rectangle has four right angles. Quadrilateral $WXYZ$ has four right angles. So $WXYZ$ is a rectangle.

(c) Every point on the perpendicular bisector of a segment is the same distance from the segment's two endpoints. Point P lies on the perpendicular bisector of $\overline{AB}$. So $PA = PB$.

4. **Given:** $\angle ABC$ is a right angle, D is in the interior of $\angle ABC$, and $m\angle ABD = m\angle DBC$. **Prove:** $m\angle ABD = 45^\circ$.

The seven statements of the proof are below, out of order.

- $2\,m\angle ABD = 90^\circ$
- $m\angle ABD + m\angle DBC = m\angle ABC$
- $m\angle ABC = 90^\circ$
- $m\angle ABD = 45^\circ$
- $m\angle ABD + m\angle ABD = 90^\circ$
- $m\angle ABD = m\angle DBC$
- $\angle ABC$ is a right angle

(a) Put the statements in order and give the reason for each.

(b) Change only the first given, to "$m\angle ABC = 130^\circ$". Which lines of the proof change, and what does the proof now conclude?

5. M is the midpoint of $\overline{AB}$, and N is the midpoint of $\overline{MB}$.

(a) If $AB = 20$, find AN, giving a reason for each step.

(b) Find a formula for AN in terms of AB that holds for every segment $\overline{AB}$, and prove it.

(c) Maya draws the figure carefully and measures $AB = 10$ cm and $AN = 7.3$ cm. Does her measurement cast doubt on your answer to (b)? Explain.

Free sample from math.wiki for classroom use. Do not post. math.wiki

6. Two lines cross at O and form four angles, $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$, in that order around O. So each angle forms a linear pair with the angle before it and the angle after it.

Given: $\angle 1 \cong \angle 2$.

Prove: all four angles are right angles.

Write a two-column proof. Do not use the Vertical Angles Theorem.

Statements	Reasons

7. Take the angle sum theorem as already proved: the three angles of any triangle add to 180°. A ray **bisects** an angle when it splits the angle into two angles of equal measure.

In $\triangle ABC$, the ray that bisects $\angle ABC$ and the ray that bisects $\angle ACB$ meet at a point I inside the triangle.

(a) Three such triangles are drawn and measured.

Triangle	$m\angle A$	$m\angle BIC$
1	40°	110°
2	70°	125°
3	100°	140°

State a conjecture that gives $m\angle BIC$ in terms of $m\angle A$. What kind of reasoning produced it?

(b) Prove your conjecture for every triangle, naming each accepted fact you use.

(c) In one such triangle, $m\angle BIC$ is three times $m\angle A$. Find $m\angle A$.

Free sample from math.wiki for classroom use. Do not post. math.wiki