Geometry Unit 1 worksheet: Inductive reasoning

Name _____ Date _____ Period _____

1. Different lines, all passing through one point P, are drawn in a plane. They cut the plane into regions.

(a) Draw 1, 2 and 3 lines through P, and count the regions each time.

(b) Maya looked only at 1 and 2 lines and conjectured that n lines make 2^n regions. Diego looked at the same two cases and conjectured $2n$ regions. Can the cases of 1 and 2 lines decide between them? What does your count for 3 lines decide?

(c) Give a reason that covers every n.

(d) How many regions do 12 lines through P make?

2. A **strip** is a row of n unit squares side by side. A rectangle in the strip is made of one or more neighboring whole squares of the strip; a single square counts as a rectangle.

(a) Count the rectangles in strips of 1, 2, 3 and 4 squares.

(b) Use the differences between your counts to predict the count for a strip of 5 squares.

(c) Give a reason that covers every n, and use it to write the count as a formula in n. Does it agree with your prediction?

(d) How many rectangles are in a strip of 30 squares?

3. Claim: if one pair of opposite sides of a quadrilateral is parallel and the other pair of opposite sides is equal in length, then the quadrilateral is a parallelogram.

Decide whether each quadrilateral is a counterexample to the claim. Show every fact you use exactly, from the coordinates.

(a) $A(0,0)$, $B(8,0)$, $C(6,3)$, $D(2,3)$

(b) $E(0,0)$, $F(6,0)$, $G(8,3)$, $H(2,3)$

(c) $J(0,0)$, $K(6,0)$, $L(9,4)$, $M(0,5)$

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4. Claim: a quadrilateral with two right angles is a rectangle. (A rectangle is a quadrilateral with four right angles.)

(a) Give a counterexample figure by coordinates, and show exactly that it meets the hypothesis and is not a rectangle.

(b) Owen repairs the claim: a quadrilateral with right angles at two **opposite** corners is a rectangle. Test his repair on the convex quadrilateral $P(0,0)$, $Q(1,-2)$, $R(5,0)$, $S(1,2)$.

(c) Repair the claim a different way so that it is true, and prove your repair. You may use that the four angles of a convex quadrilateral add to $360°$.

5. Claim: the two diagonals of every convex quadrilateral cut it into four triangles of equal area.

Maya checked a square, a rectangle and a rhombus, and in each one the claim held.

(a) Name a property all three of her figures have that the claim did not ask for.

(b) The quadrilateral $A(0,0)$, $B(6,0)$, $C(3,3)$, $D(0,3)$ is convex. Find the point O where its diagonals meet, and then the areas of triangles OAB, OBC, OCD and ODA. What does this show?

(c) Repair the claim by adding a hypothesis taken from your answer to (a).

6. The **midpoint triangle** of a triangle is the triangle whose corners are the midpoints of its three sides.

(a) For each triangle, find its area and the area of its midpoint triangle.

(i) $A(0,0)$, $B(8,0)$, $C(2,6)$

(ii) $A(0,0)$, $B(6,0)$, $C(0,10)$

(iii) $A(0,0)$, $B(4,0)$, $C(10,6)$

(b) State a conjecture about every triangle. Why are these three triangles a better test than three equilateral triangles would be?

(c) Prove your conjecture for the triangle $A(0,0)$, $B(b,0)$, $C(c,d)$ with $b>0$ and $d>0$, keeping b, c and d as letters.

(d) Explain why (c) covers every triangle, and not only some.

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7. Claim: if the three sides of a triangle have whole-number lengths, then its area is a whole number.

(a) Mateo tests three triangles: a right triangle with legs 3 and 4 and hypotenuse 5, a right triangle with legs 6 and 8 and hypotenuse 10, and the triangle with sides 5, 5 and 6. Find each area.

(b) Show exactly that the triangle with all three sides of length 2 is a counterexample.

(c) Mateo repairs the claim: if the sides of a triangle are whole numbers and not all equal, then its area is a whole number. Test the repair on the triangle with sides 2, 2 and 3, and say what was wrong with this way of repairing.

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