Geometry Unit 1 worksheet: Statements and negation (answer key)

ANSWERS AT A GLANCE

1. (a) statement, true (b) not a statement (an instruction) (c) statement, false (d) statement, true (e) not a statement (an opinion) (f) not a statement (x is not fixed) (g) not a statement (a question) (h) statement, true (i) statement, true
2. (a) "The midpoint ... is not (5, 2)." Statement true, negation false. (b) "The points ... are not collinear." Statement false, negation true. (c) "An angle that measures 90° is not acute." Statement false, negation true. (d) "The segment ... does not have length 10." Statement true, negation false. (e) "The point (4, −1) does not lie on the line." Statement true, negation false.
3. P is true and Q is false. (a) F (b) T (c) T (d) T (e) F
4. (a) No: at 90° both sentences are false. (b) $m\angle B < 90^\circ$; negation $m\angle B \geq 90^\circ$. (c) $m\angle B > 90^\circ$; negation $m\angle B \leq 90^\circ$. (d) $m\angle B = 90^\circ$; negation $m\angle B \neq 90^\circ$, an angle that is acute or obtuse.
5. (a) Both are false. (b) A statement and its negation never have the same truth value. (c) "M is not the midpoint of $\overline{AB}$."
6. (a) $P \wedge \neg Q$ (b) $P \vee \neg Q$

WORKED SOLUTIONS

1. Decide whether each sentence is a statement. If it is, say whether it is true or false. If it is not, say why.

(a) Every square is a rectangle.

(b) Draw the perpendicular bisector of $\overline{AB}$.

(c) A triangle can have two right angles.

(d) The slope of the line through (1, 2) and (3, 8) is 3.

(e) Equilateral triangles are the most beautiful triangles.

(f) The point $(x, 7)$ lies on the line $y = 2x + 1$.

(g) Are the points (0, 0), (1, 1) and (2, 2) collinear?

(h) $\angle 1$ and $\angle 2$ are supplementary when $m\angle 1 = 65^\circ$ and $m\angle 2 = 115^\circ$.

(i) Through any two points there is exactly one line.

ANSWER

(a) statement, true (b) not a statement (an instruction) (c) statement, false (d) statement, true (e) not a statement (an opinion) (f) not a statement (x is not fixed) (g) not a statement (a question) (h) statement, true (i) statement, true

SOLUTION

(a) A statement. A rectangle is a quadrilateral with four right angles, and every square has four right angles, so it is true.

(b) Not a statement. It gives an instruction; it claims nothing that could be true or false.

(c) A statement. The three angles of a triangle add to 180°, and two right angles already use all 180°, leaving 0° for the third angle. So it is false.

(d) A statement. The slope is $\frac{8-2}{3-1} = \frac{6}{2} = 3$, so it is true.

(e) Not a statement. It records a taste, and no fact settles it.

(f) Not a statement. x has not been fixed: with $x = 3$ the point (3, 7) is on the line, since 2(3) + 1 = 7, and with any other x it is not. Without a value the sentence is neither true nor false.

(g) Not a statement. It asks a question rather than making a claim. (The claim "(0, 0), (1, 1) and (2, 2) are collinear" would be a true statement.)

(h) A statement, with the angle measures fixed. 65° + 115° = 180°, so the angles are supplementary and it is true.

(i) A statement. It is a postulate of the course, so it is true.

2. Write the negation of each statement. Then give the truth value of the statement and of its negation.

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(a) The midpoint of the segment from (2, 5) to (8, −1) is (5, 2).

(b) The points (1, 2), (3, 6) and (4, 9) are collinear.

(c) An angle that measures 90° is acute.

(d) The segment from (−3, 1) to (3, 9) has length 10.

(e) The point (4, −1) lies on the line $y = -2x + 7$.

ANSWER

(a) "The midpoint ... is not (5, 2)." Statement true, negation false. (b) "The points ... are not collinear." Statement false, negation true. (c) "An angle that measures 90° is not acute." Statement false, negation true. (d) "The segment ... does not have length 10." Statement true, negation false. (e) "The point (4, −1) does not lie on the line." Statement true, negation false.

SOLUTION

(a) Negation: "The midpoint of the segment from (2, 5) to (8, −1) is not (5, 2)." The midpoint is $\left(\frac{2+8}{2}, \frac{5+(-1)}{2}\right) = (5, 2)$, so the statement is true and its negation is false.

(b) Negation: "The points (1, 2), (3, 6) and (4, 9) are not collinear." The slope from (1, 2) to (3, 6) is $\frac{4}{2} = 2$, and from (3, 6) to (4, 9) it is $\frac{3}{1} = 3$. The slopes differ, so the points are not collinear: the statement is false and its negation is true.

(c) Negation: "An angle that measures 90° is not acute." An acute angle measures less than 90°, and 90° is not less than 90°. The statement is false and its negation is true.

(d) Negation: "The segment from (−3, 1) to (3, 9) does not have length 10." Its length is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$, so the statement is true and its negation is false.

(e) Negation: "The point (4, −1) does not lie on the line $y = -2x + 7$." Substituting $x = 4$ gives $y = -8 + 7 = -1$, so the point is on the line: the statement is true and its negation is false.

3. Let P be "the distance from (1, 1) to (4, 5) is 5", and let Q be "the midpoint of the segment from (−3, 2) to (5, 6) is (1, 3)".

First decide whether P and Q are true. Then give the truth value of each statement.

(a) $\neg P \wedge Q$ (b) $\neg(P \wedge Q)$ (c) $\neg P \vee \neg Q$

(d) $\neg(\neg P \vee Q)$ (e) $\neg(P \vee Q)$

ANSWER

P is true and Q is false. (a) F (b) T (c) T (d) T (e) F

SOLUTION

The distance from (1, 1) to (4, 5) is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$, so P is true. The midpoint of the second segment is $\left(\frac{-3+5}{2}, \frac{2+6}{2}\right) = (1, 4)$, not (1, 3), so Q is false. So $\neg P$ is false and $\neg Q$ is true.

(a) $\neg P$ is false, so $\neg P \wedge Q$ is false.

(b) $P \wedge Q$ is false because Q is false, so $\neg(P \wedge Q)$ is true.

(c) $\neg Q$ is true, so $\neg P \vee \neg Q$ is true.

(d) $\neg P$ and Q are both false, so $\neg P \vee Q$ is false and $\neg(\neg P \vee Q)$ is true.

(e) P is true, so $P \vee Q$ is true and $\neg(P \vee Q)$ is false.

4. $\angle B$ is one particular angle, with $0° < m\angle B < 180°$.

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(a) Is "$\angle B$ is obtuse" the negation of "$\angle B$ is acute"? Test the case $m\angle B = 90^\circ$ and explain.

(b) Write "$\angle B$ is acute" as an inequality about $m\angle B$, and write its negation as an inequality.

(c) Do the same for "$\angle B$ is obtuse".

(d) Do the same for "$\angle B$ is a right angle", and say in words what kind of angle the negation describes.

ANSWER

(a) No: at 90° both sentences are false. (b) $m\angle B < 90^\circ$; negation $m\angle B \geq 90^\circ$. (c) $m\angle B > 90^\circ$; negation $m\angle B \leq 90^\circ$. (d) $m\angle B = 90^\circ$; negation $m\angle B \neq 90^\circ$, an angle that is acute or obtuse.

SOLUTION

(a) No. When $m\angle B = 90^\circ$, $\angle B$ is a right angle: it is not acute (that needs less than 90°) and not obtuse (that needs more than 90°). Both sentences are false at once. A negation must be true exactly when the original is false, so "obtuse" cannot be the negation of "acute".

(b) "$\angle B$ is acute" says $m\angle B < 90^\circ$. It is false exactly when $m\angle B$ is 90° or more, so the negation is $m\angle B \geq 90^\circ$ ("$\angle B$ is right or obtuse").

(c) "$\angle B$ is obtuse" says $m\angle B > 90^\circ$. It is false exactly when $m\angle B$ is 90° or less, so the negation is $m\angle B \leq 90^\circ$ ("$\angle B$ is acute or right").

(d) "$\angle B$ is a right angle" says $m\angle B = 90^\circ$. Its negation is $m\angle B \neq 90^\circ$: within 0° to 180°, that means $\angle B$ is acute or obtuse.

5. Diego writes the negation of "M is the midpoint of $\overline{AB}$" as "M is not on $\overline{AB}$".

(a) Take $A = (0, 0)$, $B = (8, 0)$ and $M = (2, 0)$. Give the truth value of the original sentence and of Diego's sentence.

(b) Explain why part (a) shows that Diego's sentence is not the negation.

(c) Write the correct negation.

ANSWER

(a) Both are false. (b) A statement and its negation never have the same truth value. (c) "M is not the midpoint of $\overline{AB}$."

SOLUTION

(a) The midpoint of $\overline{AB}$ is $\left(\frac{0+8}{2}, 0\right) = (4, 0)$, so $M = (2, 0)$ is not the midpoint: the original is false. But M lies on $\overline{AB}$, between (0, 0) and (8, 0), so Diego's sentence is false too.

(b) The negation of a statement is true exactly when the statement is false. Here the original is false, so its negation must be true, and Diego's sentence is false. So Diego's sentence cannot be the negation. It leaves out every point that is on $\overline{AB}$ but not at its middle.

(c) "M is not the midpoint of $\overline{AB}$." It is true here, as it must be.

6. Each statement below uses the letters P and Q once each, one ¬ and one other connective. Its column is given in the usual row order P, Q = TT, TF, FT, FF. Find the statement, and show the columns of the other choices you ruled out.

(a) The connective is $\wedge$ and the column is F, T, F, F.

(b) The connective is $\vee$ and the column is T, T, F, T.

ANSWER

(a) $P \wedge \neg Q$ (b) $P \vee \neg Q$

SOLUTION

The ¬ can sit on P, on Q, or over the whole statement, so there are three choices each time.

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(a) $\neg P \wedge Q$ has column F, F, T, F. $P \wedge \neg Q$ has column F, T, F, F. $\neg(P \wedge Q)$ has column F, T, T, T. Only $P \wedge \neg Q$ (or the same thing written $\neg Q \wedge P$) matches: it is true only in row 2, where P is true and Q is false.

(b) $\neg P \vee Q$ has column T, F, T, T. $P \vee \neg Q$ has column T, T, F, T. $\neg(P \vee Q)$ has column F, F, F, T. Only $P \vee \neg Q$ (or $\neg Q \vee P$) matches: it is false only in row 3, where P is false and $\neg Q$ is false.

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