Geometry Unit 1 worksheet: Points, lines and planes (answer key)

ANSWERS AT A GLANCE

1. (a) undefined. (b) defined, from point and line. (c) defined, from point and line (with "between"). (d) undefined. (e) defined, from point and plane.
2. (a) collinear (b) not collinear (c) not collinear (d) collinear (e) not collinear
3. (a) correct. (b) not correct: it names a segment. (c) not correct: S is not on t. (d) correct. (e) not correct: P, Q, R are collinear.
4. (a) Yes; P lies between Q and R. (b) No: the longest distance is $PQ = 9$, but $PS + QS = 10$. (c) $QT = 12$ and $TR = 2$.
5. (a) Both slopes from $(a, 0)$ equal $-\dfrac{b}{a}$, so the points are collinear. (b) $c = -2$
6. $y = 1$, so the last post stands at $(15, 1)$.
7. (a) Yes. (b) No, they are noncoplanar. (c) F
8. See the proof below.

WORKED SOLUTIONS

1. Say whether each word is an undefined term or a defined term. For each defined term, name the undefined terms its definition is built on: (a) plane, (b) collinear, (c) segment, (d) point, (e) coplanar.

ANSWER

(a) undefined. (b) defined, from point and line. (c) defined, from point and line (with "between"). (d) undefined. (e) defined, from point and plane.

SOLUTION

The three undefined terms are point, line and plane, so (a) and (d) are undefined.

(b) Points are collinear when one line passes through all of them: the definition uses point and line.

(c) Segment $\overline{AB}$ is the points A and B and every point of the line $\overleftrightarrow{AB}$ between them: point, line and the relation word "between".

(e) Points are coplanar when one plane contains all of them: point and plane.

2. Use the slope test to decide whether the three points are collinear.

(a) $(-4, 9)$, $(0, 3)$, $(6, -6)$ (b) $(-1, -5)$, $(2, 1)$, $(8, 12)$ (c) $(3, 8)$, $(3, -2)$, $(5, 4)$

(d) $(-6, -2)$, $(-2, 1)$, $(10, 10)$ (e) $(0, -4)$, $(5, 0)$, $(-10, -11)$

ANSWER

(a) collinear (b) not collinear (c) not collinear (d) collinear (e) not collinear

SOLUTION

Measure both slopes from the first point.

(a) $\dfrac{3-9}{0-(-4)} = -\dfrac{3}{2}$ and $\dfrac{-6-9}{6-(-4)} = \dfrac{-15}{10} = -\dfrac{3}{2}$. Equal, so collinear.

(b) $\dfrac{1-(-5)}{2-(-1)} = 2$ and $\dfrac{12-(-5)}{8-(-1)} = \dfrac{17}{9}$. Different, so not collinear.

(c) The first two points share $x = 3$, so the only line through them is vertical. The third point has $x = 5$, so it is off that line: not collinear.

(d) $\dfrac{1-(-2)}{-2-(-6)} = \dfrac{3}{4}$ and $\dfrac{10-(-2)}{10-(-6)} = \dfrac{12}{16} = \dfrac{3}{4}$. Equal, so collinear.

(e) $\dfrac{0-(-4)}{5-0} = \dfrac{4}{5}$ and $\dfrac{-11-(-4)}{-10-0} = \dfrac{7}{10}$. Different, so not collinear. (The line through the first two points reaches $y = -12$ at $x = -10$, one unit below the third point.)

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3. Line t passes through points P, Q and R. Point S is not on t. All four points lie in plane $\mathcal{K}$. Say whether each is a correct name. If it is not, say why.

(a) $\overleftrightarrow{RP}$ for line t

(b) $\overline{PQ}$ for line t

(c) $\overleftrightarrow{QS}$ for line t

(d) plane QSR for plane $\mathcal{K}$

(e) plane PQR for plane $\mathcal{K}$

ANSWER

(a) correct. (b) not correct: it names a segment. (c) not correct: S is not on t. (d) correct. (e) not correct: P, Q, R are collinear.

SOLUTION

(a) Correct. Any two points of a line name it, in either order.

(b) Not correct. $\overline{PQ}$ is the segment from P to Q, which stops at its endpoints. A line goes on without end, and its name carries the double arrow.

(c) Not correct. S is not on t, so $\overleftrightarrow{QS}$ is a different line through Q.

(d) Correct. Q, S and R lie in $\mathcal{K}$, and they are noncollinear, since S is off t, the only line through Q and R. Exactly one plane holds three noncollinear points, so plane QSR is $\mathcal{K}$.

(e) Not correct. P, Q and R all lie on t, so they are collinear. A plane is named by three of its points that do not all lie on one line; three collinear points do not pick out one plane.

4. Elena's hiking map marks four campsites P, Q, R and S, and lists the straight-line distances between some of them, in kilometers: $PQ = 9$, $QR = 14$, $PR = 5$, $PS = 6$ and $QS = 4$.

(a) Do P, Q and R lie on one straight line? If they do, which campsite lies between the other two?

(b) Do P, Q and S lie on one straight line? Explain.

(c) A fifth campsite T lies on $\overline{QR}$ with $PT = 3$ and T between P and R. Find QT and TR.

ANSWER

(a) Yes; P lies between Q and R. (b) No: the longest distance is $PQ = 9$, but $PS + QS = 10$. (c) $QT = 12$ and $TR = 2$.

SOLUTION

Use the distance test: three points are collinear exactly when the longest of their three distances equals the sum of the other two.

(a) The longest distance is $QR = 14$, and $PQ + PR = 9 + 5 = 14$. So P, Q and R are collinear, and P, the point not at an end of $\overline{QR}$, lies between Q and R.

(b) The longest distance is $PQ = 9$, but $PS + QS = 6 + 4 = 10$, which is more than 9. The way from P to S to Q bends at S, so the three campsites are the corners of a triangle, not points on one line.

(c) Along the line the order is Q, P, T, R: P is between Q and R, and T is between P and R. So the pieces add up:

$$QT = QP + PT = 9 + 3 = 12, \qquad TR = PR - PT = 5 - 3 = 2$$

Check: $QT + TR = 12 + 2 = 14 = QR$.

5. Let a and b be any numbers that are not 0.

(a) Show that the points $(a, 0)$, $(0, b)$ and $(2a, -b)$ are collinear, whatever a and b are.

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(b) Find the number c for which $(a, 0)$, $(0, b)$ and $(3a, cb)$ are collinear for every such a and b.

ANSWER

(a) Both slopes from $(a, 0)$ equal $-\dfrac{b}{a}$, so the points are collinear. (b) $c = -2$

SOLUTION

(a) Measure both slopes from the shared point $(a, 0)$. The x-coordinates 0 and $2a$ both differ from a, since $a \neq 0$, so both slopes exist. From $(a, 0)$ to $(0, b)$:

$$\frac{b - 0}{0 - a} = -\frac{b}{a}$$

From $(a, 0)$ to $(2a, -b)$:

$$\frac{-b - 0}{2a - a} = -\frac{b}{a}$$

The slopes are equal and share the point $(a, 0)$, so the three points are collinear for every nonzero a and b.

(b) The slope from $(a, 0)$ to $(3a, cb)$ is $\dfrac{cb - 0}{3a - a} = \dfrac{cb}{2a}$. It must equal $-\dfrac{b}{a}$:

$$\begin{aligned} \frac{cb}{2a} &= -\frac{b}{a} \\ cb &= -2b \\ c &= -2 \end{aligned}$$

dividing by b, which is not 0. **Check.** From $(0, b)$ to $(3a, -2b)$ the slope is $\dfrac{-3b}{3a} = -\dfrac{b}{a}$, the same again.

6. Hannah is putting up a straight fence across a garden. On a plan marked in meters, the first two posts stand at $(-1, -3)$ and $(3, -2)$. The last post must stand where $x = 15$, in line with the first two. Find the y-coordinate of the last post.

ANSWER

$y = 1$, so the last post stands at (15, 1).

SOLUTION

Three posts in a straight fence are collinear, so the slope from the first post to the last must equal the slope from the first post to the second. From the first post to the second the fence goes 4 across and 1 up, a slope of $\dfrac{1}{4}$.

From the first post to $x = 15$ is 15 − (−1) = 16 across, which is 4 times 4. With the same slope the fence must rise 4 times 1, which is 4. So $y = -3 + 4 = 1$.

Check: the slope from (−1, −3) to (15, 1) is $\dfrac{4}{16} = \dfrac{1}{4}$, the same as before.

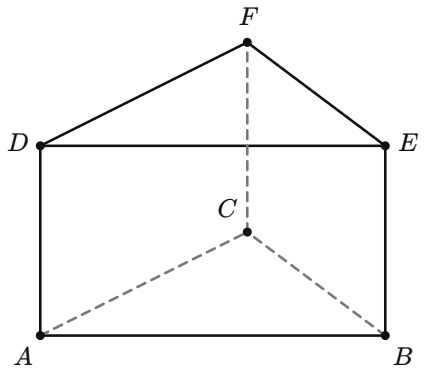
7. The figure shows a triangular prism. Its bottom face is triangle ABC and its top face is triangle DEF, with D directly above A, E above B and F above C.

(a) Are A, B and F coplanar? Explain.

(b) Are A, C, D and E coplanar? Explain.

(c) Name the one other labeled point that is coplanar with B, C and E.

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ANSWER

(a) Yes. (b) No, they are noncoplanar. (c) F

SOLUTION

(a) Yes. Any three points are coplanar. A, B and F are noncollinear (F is off the bottom face, so it is off $\overleftrightarrow{AB}$), so exactly one plane holds them, plane ABF. It is not a face of the prism; it slices through it.

(b) No. A, C and D are noncollinear, so exactly one plane holds them: the plane of the side face $ACFD$. E sits above B, and that face does not reach B or E, so E is not in the plane. The four points are noncoplanar.

(c) B, C and E are noncollinear, so they lie in exactly one plane, the side face $BCFE$. Of the other labeled points, F is a corner of that face, while A and D are off it. The answer is F.

8. Use only these rules.

- **Rule 1.** Through any two different points there is exactly one line.
- **Rule 2.** Every line contains at least two points.
- **Rule 3.** There are three points that do not all lie on one line.

Given: a line ℓ and a point P that is not on ℓ. Prove that at least two different lines pass through P and meet ℓ.

ANSWER

Proof below: the lines $\overleftrightarrow{PX}$ and $\overleftrightarrow{PY}$, for two different points X and Y of ℓ.

SOLUTION

By Rule 2, ℓ contains two different points; call them X and Y. P is not on ℓ, so $P \neq X$ and $P \neq Y$.

By Rule 1 there is a line through P and X, $\overleftrightarrow{PX}$, and a line through P and Y, $\overleftrightarrow{PY}$. Both pass through P, and they meet ℓ, at X and at Y.

Suppose they were the same line. Then that line would pass through X and Y. By Rule 1 only one line passes through the two different points X and Y, and ℓ is such a line, so the line would be ℓ. But it passes through P, and P is not on ℓ. This is impossible, so $\overleftrightarrow{PX}$ and $\overleftrightarrow{PY}$ are different lines, and at least two different lines pass through P and meet ℓ.

SCORING

1 point: uses Rule 2 to get two different points X and Y on ℓ, and Rule 1 to get the lines $\overleftrightarrow{PX}$ and $\overleftrightarrow{PY}$. 1 point: notes both lines pass through P and meet ℓ. 2 points: shows the two lines are different (if equal, Rule 1 makes the line ℓ, which does not hold P).

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