Geometry Unit 1 worksheet: Axioms, definitions and theorems

Name _____ Date _____ Period _____

1. Call a segment in the coordinate plane **neat** if its length is a whole number.

(a) Use the distance formula to decide whether the segment from $(-2, 3)$ to $(4, -5)$ is neat.

(b) Decide whether the segment from $(0, 0)$ to $(2, 3)$ is neat.

(c) Decide whether the segment from $(0, 0)$ to $(\sqrt{7}, 3)$ is neat.

(d) Show that for every positive integer n, the segment from (n, n) to $(4n, 5n)$ is neat, and give its length.

2. Call an integer a **near-square** if it equals $k^2 + k$ for some integer k.

(a) Decide whether 56 is a near-square and whether 40 is a near-square.

(b) Show that if N is a near-square, then $4N + 1$ is the square of an integer.

(c) Show that every near-square is even. (Recall: an integer is odd if it equals $2m + 1$ for some integer m, and every integer is even or odd.)

3. A system has two axioms about points and lines.

S1. Every line has exactly three points on it.

S2. Any two different points lie on exactly one line.

A world obeys S1 and S2 and has exactly 13 points.

(a) How many lines pass through a given point P?

(b) How many lines does the world have?

4. Add a third axiom to S1 and S2.

S1. Every line has exactly three points on it.

S2. Any two different points lie on exactly one line.

S3. There are at least two lines.

(a) Show that S1 cannot be proved from S2 and S3: describe a world, giving its points and its lines, in which S2 and S3 are true and S1 is false.

(b) Show in the same way that S2 cannot be proved from S1 and S3.

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5. A short text proves six results. Each line says what its proof uses.

- Result 1 is proved from Axioms A and B.
- Result 2 is proved from Result 1 and a definition.
- Result 3 is proved from Axiom C alone.
- Result 4 is proved from Results 2 and 3.
- Result 5 is proved from Result 4 in two lines.
- Result 6 is proved from Results 1 and 3.

(a) A reader finds a gap in the proof of Result 2. Which results are left without a proof until the gap is mended, and which keep their proofs?

(b) Which of the words theorem, lemma or corollary best suits Result 5, and of which result?

(c) The author wants to rewrite the proof of Result 3 so that it cites Result 5. Explain why that is not allowed.

6. Here is a world for the axioms S1 and S2 of the previous problem. It has nine points, named 1 to 9, and these twelve lines:

$\{1,2,3\}$, $\{4,5,6\}$, $\{7,8,9\}$, $\{1,4,7\}$, $\{2,5,8\}$, $\{3,6,9\}$, $\{1,5,9\}$, $\{2,6,7\}$, $\{3,4,8\}$, $\{3,5,7\}$, $\{1,6,8\}$, $\{2,4,9\}$

(a) Name the line through the points 2 and 6, and the line through the points 4 and 9.

(b) List the lines through the point 5. Does their number agree with what S1 and S2 force for a world with nine points?

(c) Find two different lines with no point in common. What does this show about the statement "any two different lines have a point in common", for a system whose only axioms are S1 and S2?

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7. Take the angle sum theorem as already proved: the three angles of any triangle add to 180°. An angle is **obtuse** if its measure is greater than 90° and less than 180°.

(a) Prove that a triangle has at most one obtuse angle.

(b) Why is the result of (a) usually called a corollary rather than a theorem?

(c) The four angles of a convex quadrilateral add to 360°. Use the same kind of argument to prove that a convex quadrilateral cannot have four obtuse angles. Then explain why this argument does not show that a convex quadrilateral has at most one obtuse angle.

Statements	Reasons

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