Triangle congruence criteria: SSS, SAS, ASA and AAS

Two triangles are congruent when all six pairs of parts match. Which three matching pairs are enough to prove all six?

WORDS TO KNOW

Included angle: the angle between two sides, at the vertex they share.

Included side: the side that joins the vertices of two angles.

Congruence criterion: a list of three pairs of parts that, once they match, force the other three pairs to match.

S and A: S stands for a pair of sides, A for a pair of angles, read in order around the triangle.

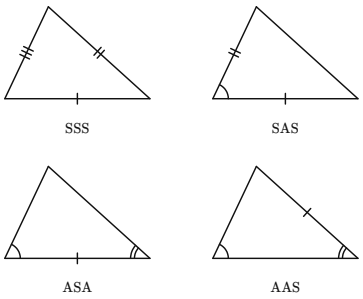
Postulate: a fact accepted without proof as a starting point.

Reflexive property: a segment or angle is congruent to itself. Use it for a side two triangles share.

KEY IDEAS

1. Four ways to prove two triangles congruent

A triangle is rigid: once a few parts are fixed, the rest have no freedom. Each criterion names three parts that fix the triangle.



SSS: three pairs of sides.

SAS: two pairs of sides and the angle **between** them.

ASA: two pairs of angles and the side **between** them.

AAS: two pairs of angles and a side that is **not** between them.

SSS, SAS and ASA are postulates. AAS is a theorem: two pairs of angles match, so the third pair matches too (the angles of a triangle add to 180°). Then the side sits between two matched angles, and that is ASA.

2. The parts must correspond

Each pair matches a part of one triangle with the part in the same position in the other. The congruence statement lists the vertices in that same order. Once the triangles are proved congruent, the three pairs you were never given are congruent too.

Included means between. Check where each marked angle sits before you name the criterion.

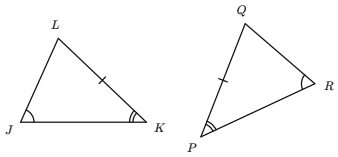
WATCH OUT

- **SSA is not a criterion.** Two sides and an angle that is not between them can fit two different triangles. Example: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\angle A \cong \angle D$ is SSA, because the angle between $\overline{AB}$ and $\overline{BC}$ is $\angle B$.
- **Three angles are never enough.** AAA fixes the shape but not the size: one triangle can be a smaller copy of the other.
- **Match the marks, not the positions on the page.** The second triangle may be turned or flipped over.
- **Parts must be known, not seen.** Each pair must be given, marked, or proved on an earlier line. Parts that look equal count for nothing.

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WORKED EXAMPLES

Example 1. Which criterion proves these triangles congruent? Write the congruence statement.

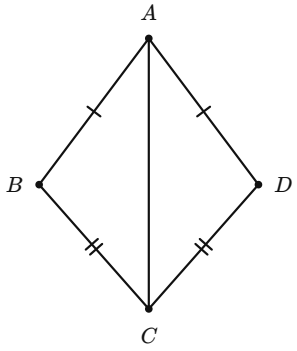


1. Match the marks. One arc: $\angle J$ and $\angle R$. Two arcs: $\angle K$ and $\angle P$. One tick: $\overline{KL}$ and $\overline{PQ}$.
2. That is two angles and a side. The side between $\angle J$ and $\angle K$ is $\overline{JK}$, but the ticked side is $\overline{KL}$. So the side is not included: AAS.
3. The sides must correspond. $\overline{KL}$ faces $\angle J$, and $\overline{PQ}$ faces $\angle R$, the partner of $\angle J$. They do.

Answer: $\triangle JKL \cong \triangle RPQ$ by AAS, with $J \leftrightarrow R$, $K \leftrightarrow P$, $L \leftrightarrow Q$.

Example 2. In kite $ABCD$, $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$. Prove $\triangle ABC \cong \triangle ADC$.

Plan: two pairs of sides are given, and the diagonal $\overline{AC}$ is a side of both triangles, so it is the third pair.



Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	given
2. $\overline{CB} \cong \overline{CD}$	given
3. $\overline{AC} \cong \overline{AC}$	reflexive property of congruence
4. $\triangle ABC \cong \triangle ADC$	SSS postulate: lines 1, 2 and 3

Example 3. $AB = DE = 11$, $m\angle B = 58°$, $m\angle E = (5x - 7)°$, $BC = 9$ and $EF = 2x - 17$. Find the x that makes SAS prove $\triangle ABC \cong \triangle DEF$.

1. $\angle B$ is between $\overline{AB}$ and $\overline{BC}$, and $\angle E$ is between $\overline{DE}$ and $\overline{EF}$. So SAS needs the angles to match and the second sides to match.
2. Angles: $5x - 7 = 58$, so $5x = 65$ and $x = 13$.
3. Sides: $2x - 17 = 9$, so $2x = 26$ and $x = 13$.
4. Both give the same value, so SAS applies.

Answer: $x = 13$. **Check:** 5(13) − 7 = 58 and 2(13) − 17 = 9.

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