Geometry Unit 4 worksheet: SSS, SAS, ASA, AAS (answer key)

ANSWERS AT A GLANCE

1. (a) $\angle S \cong \angle V$. (b) $\overline{RT} \cong \overline{UW}$. (c) ASA and AAS each need two pairs of angles, and one added pair supplies at most one. $\angle R$ and $\angle T$ are not between the two given sides, so either one gives SSA, which is not a criterion.
2. (a) $x = 6$ and $y = 5$, or $x = 6$ and $y = 1$. (b) For $x = 6$, $y = 5$: $\triangle ABC \cong \triangle DEF$. For $x = 6$, $y = 1$: $\triangle ABC \cong \triangle FED$.
3. (a) $\triangle PQR \cong \triangle ZXY$ by SAS. (b) $\triangle PQR \cong \triangle XZY$ by ASA. (c) $\triangle PQR \cong \triangle ZYX$ by AAS. (d) No criterion applies: the sides do not correspond.
4. (a) $\triangle SAB \cong \triangle DCB$ by ASA. (b) $AS = 230$ m.
5. (a) Yes, by SSS: the third sides must be equal too. (b) One possible pair: sides 5, 6, 7 and sides 5, 5, 8.
6. See the proof below.

WORKED SOLUTIONS

1. In $\triangle RST$ and $\triangle UVW$, $\overline{RS} \cong \overline{UV}$ and $\overline{ST} \cong \overline{VW}$. You may add one more pair of corresponding parts.

(a) Which pair of angles would let SAS prove the triangles congruent?

(b) Which pair of sides would let SSS prove it?

(c) Explain why no single added pair could complete ASA or AAS, and why adding $\angle R \cong \angle U$ or $\angle T \cong \angle W$ would not prove the triangles congruent.

ANSWER

(a) $\angle S \cong \angle V$. (b) $\overline{RT} \cong \overline{UW}$. (c) ASA and AAS each need two pairs of angles, and one added pair supplies at most one. $\angle R$ and $\angle T$ are not between the two given sides, so either one gives SSA, which is not a criterion.

SOLUTION

(a) $\overline{RS}$ and $\overline{ST}$ meet at S, so their included angle is $\angle S$; in the other triangle $\overline{UV}$ and $\overline{VW}$ meet at V. SAS needs $\angle S \cong \angle V$.

(b) The third sides are $\overline{RT}$ and $\overline{UW}$, so SSS needs $\overline{RT} \cong \overline{UW}$.

(c) ASA and AAS each use two pairs of angles and one pair of sides. Two pairs of sides are already given and no pair of angles, so one more pair can bring the angle count to one at most: neither criterion can be reached.

The other two angles fail for a different reason. $\angle R$ lies between $\overline{RS}$ and $\overline{RT}$, and $\angle T$ between $\overline{ST}$ and $\overline{TR}$; neither lies between the two given sides $\overline{RS}$ and $\overline{ST}$. Read around the triangle, either choice gives side, side and an angle that is not included: SSA, which is not a criterion. The triangles may still be congruent, but those three facts do not force it. So $\angle S \cong \angle V$ and $\overline{RT} \cong \overline{UW}$ are the only single pairs that work.

2. In $\triangle ABC$, $AB = x + y$, $BC = 2x - y$ and $CA = 10$. In $\triangle DEF$, $DE = 11$, $EF = 7$ and $FD = 10$. No pairing of the vertices is given.

(a) Find every pair of values of x and y for which SSS proves the two triangles congruent.

(b) For each pair, write the congruence statement that begins $\triangle ABC \cong$, with the vertices in matching order.

ANSWER

(a) $x = 6$ and $y = 5$, or $x = 6$ and $y = 1$. (b) For $x = 6$, $y = 5$: $\triangle ABC \cong \triangle DEF$. For $x = 6$, $y = 1$: $\triangle ABC \cong \triangle FED$.

SOLUTION

(a) The sides of $\triangle DEF$ are 11, 7 and 10, all different. $\overline{CA}$ has length 10, so it can only match $\overline{FD}$. Then AB and BC must be 11 and 7 in some order, which gives two cases.

Case 1. $AB = 11$ and $BC = 7$:

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$$x + y = 11 \qquad 2x - y = 7$$

Adding the equations, $3x = 18$, so $x = 6$, and then $y = 11 - 6 = 5$.

Case 2. $AB = 7$ and $BC = 11$:

$$x + y = 7 \qquad 2x - y = 11$$

Adding, $3x = 18$, so $x = 6$, and then $y = 7 - 6 = 1$.

Check. Case 1: $AB = 6 + 5 = 11$ and $BC = 2(6) - 5 = 7$. Case 2: $AB = 6 + 1 = 7$ and $BC = 2(6) - 1 = 11$. Both give positive lengths.

(b) Match each vertex by the two sides that meet there.

Case 1: $\overline{AB}$ and $\overline{CA}$ match $\overline{DE}$ and $\overline{FD}$, which meet at D, so $A \leftrightarrow D$; likewise $B \leftrightarrow E$ and $C \leftrightarrow F$: $\triangle ABC \cong \triangle DEF$.

Case 2: $\overline{AB}$ (7) matches $\overline{EF}$ and $\overline{CA}$ matches $\overline{FD}$, which meet at F, so $A \leftrightarrow F$. $\overline{AB}$ and $\overline{BC}$ (7 and 11) match $\overline{EF}$ and $\overline{DE}$, which meet at E, so $B \leftrightarrow E$, and $C \leftrightarrow D$: $\triangle ABC \cong \triangle FED$.

3. Each list gives three pairs of congruent parts of $\triangle PQR$ and $\triangle XYZ$. The pairs are not always in letter order. For each list, write a congruence statement that begins $\triangle PQR \cong$ with the vertices in matching order and name the criterion, or say that no criterion applies and why.

(a) $\overline{PQ} \cong \overline{ZX}$, $\angle Q \cong \angle X$, $\overline{QR} \cong \overline{XY}$

(b) $\angle R \cong \angle Y$, $\overline{RP} \cong \overline{YX}$, $\angle P \cong \angle X$

(c) $\angle P \cong \angle Z$, $\angle Q \cong \angle Y$, $\overline{QR} \cong \overline{YX}$

(d) $\angle P \cong \angle Z$, $\angle Q \cong \angle Y$, $\overline{PR} \cong \overline{YX}$

ANSWER

(a) $\triangle PQR \cong \triangle ZXY$ by SAS. (b) $\triangle PQR \cong \triangle XZY$ by ASA. (c) $\triangle PQR \cong \triangle ZYX$ by AAS. (d) No criterion applies: the sides do not correspond.

SOLUTION

Match the vertices first, then check where the side or angle sits.

(a) The angles give $Q \leftrightarrow X$. $\overline{PQ}$ matches $\overline{ZX}$, so $P \leftrightarrow Z$, and then $R \leftrightarrow Y$. $\angle Q$ is included between $\overline{PQ}$ and $\overline{QR}$, and $\angle X$ between $\overline{ZX}$ and $\overline{XY}$: $\triangle PQR \cong \triangle ZXY$ by SAS.

(b) $R \leftrightarrow Y$ and $P \leftrightarrow X$, so $Q \leftrightarrow Z$. $\overline{RP}$ joins the vertices of $\angle R$ and $\angle P$, and $\overline{YX}$ joins those of $\angle Y$ and $\angle X$: $\triangle PQR \cong \triangle XZY$ by ASA.

(c) $P \leftrightarrow Z$ and $Q \leftrightarrow Y$, so $R \leftrightarrow X$. $\overline{QR}$ is not between $\angle P$ and $\angle Q$; it faces $\angle P$. Under the pairing it matches $\overline{YX}$, which faces $\angle Z$, the angle matched with $\angle P$. So the side corresponds: $\triangle PQR \cong \triangle ZYX$ by AAS.

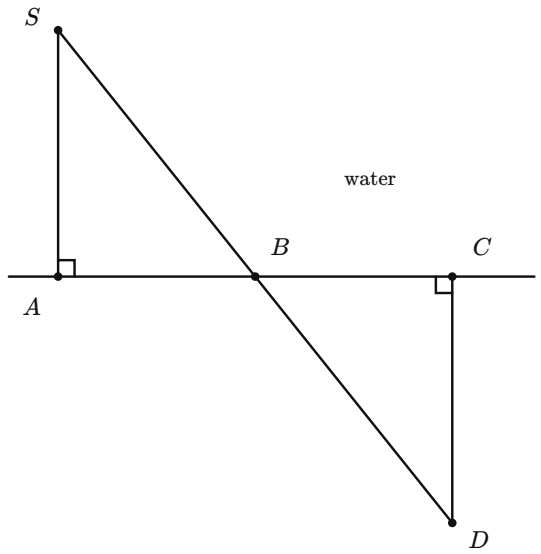
(d) The angles again force $P \leftrightarrow Z$, $Q \leftrightarrow Y$, $R \leftrightarrow X$, so $\overline{PR}$ corresponds to $\overline{ZX}$, not to $\overline{YX}$. $\overline{PR}$ faces $\angle Q$, but $\overline{YX}$ faces $\angle Z$, which is matched with $\angle P$. The two sides sit in different places, so AAS does not apply, and no criterion does.

4. Ngozi stands at A on a straight shore and sights a ship S, so that $\overline{AS}$ is perpendicular to the shore. She walks along the shore to a point B and plants a post, then walks on the same distance to C, so that $BC = AB$. At C she turns away from the water and walks perpendicular to the shore until she reaches the point D where the post at B lies exactly in line with the ship. She measures $CD = 230$ m.

(a) Name two congruent triangles, the criterion that proves it, and the three pairs of parts it uses.

(b) How far is the ship from A?

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ANSWER
(a) $\triangle SAB \cong \triangle DCB$ by ASA. (b) $AS = 230$ m.

SOLUTION
(a) Compare $\triangle SAB$, reaching out over the water, with $\triangle DCB$, on land.

- $\angle SAB \cong \angle DCB$: both are right angles, because both segments are perpendicular to the shore, and all right angles are congruent.
- $\overline{AB} \cong \overline{CB}$, because she paced BC to equal AB.
- $\angle ABS \cong \angle CBD$: A, B, C lie on the shore and S, B, D lie on the sight line, so these are vertical angles.

$\overline{AB}$ joins the vertices of $\angle SAB$ and $\angle ABS$, and $\overline{CB}$ joins those of $\angle DCB$ and $\angle CBD$, so the side is included and $\triangle SAB \cong \triangle DCB$ by ASA, with $S \leftrightarrow D$, $A \leftrightarrow C$, $B \leftrightarrow B$.

(b) Under that pairing $\overline{AS}$ corresponds to $\overline{CD}$, and corresponding parts of congruent triangles are congruent. So $AS = CD = 230$ m, found without leaving the land.

5. (a) Liam says: "If two triangles have two pairs of congruent sides and the same perimeter, then they are congruent." Is he right? Explain, naming a criterion.

(b) Liam then says that one pair of congruent sides and the same perimeter is enough. Show that he is wrong by giving the side lengths of two triangles that are not congruent, each with a side of 5 and a perimeter of 18.

ANSWER
(a) Yes, by SSS: the third sides must be equal too. (b) One possible pair: sides 5, 6, 7 and sides 5, 5, 8.

SOLUTION
(a) Yes. Call the matching sides a and b in both triangles, and the third sides c and c'. Equal perimeters say $a + b + c = a + b + c'$, so $c = c'$. Now all three pairs of sides are congruent, and SSS proves the triangles congruent.

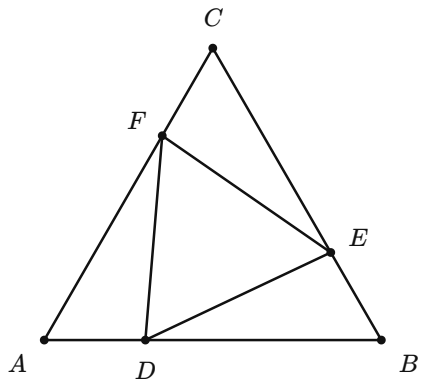
(b) Take sides 5, 6, 7 and sides 5, 5, 8. Both have a side of 5, and both perimeters are 18. Each is a real triangle, since in each the two shorter sides add to more than the longest (5 + 6 > 7 and 5 + 5 > 8). The lists of sides differ, so the triangles cannot be congruent. Other pairs work as well; the point is that two remaining sides with a fixed sum can be split in many ways.

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6. **Given.** In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$ and $\angle A \cong \angle B$. Points D, E and F lie on the sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, each between the two vertices, with $\overline{AD} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CF}$. Segments $\overline{DE}$, $\overline{EF}$ and $\overline{FD}$ are drawn.

Prove. $\triangle ADF \cong \triangle BED$.

Write a two-column proof.



ANSWER

$\triangle ADF \cong \triangle BED$ by SAS: $\overline{AD} \cong \overline{BE}$ (given), $\angle A \cong \angle B$ (given), and $\overline{AF} \cong \overline{BD}$, found by taking $AD = CF$ away from $AB = AC$.

SOLUTION

Plan. One pair of sides, $\overline{AD}$ and $\overline{BE}$, and the angles at A and B are given. The second pair of sides, $\overline{AF}$ and $\overline{BD}$, is not: each is what is left of a congruent side, $\overline{AC}$ or $\overline{AB}$, once a congruent piece, $\overline{CF}$ or $\overline{AD}$, is cut off.

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	given
2. $AB = AC$	definition of congruent segments: line 1
3. $\overline{AD} \cong \overline{CF}$	given
4. $AD = CF$	definition of congruent segments: line 3
5. $AD + DB = AB$	segment addition postulate: D is between A and B
6. $CF + FA = AC$	segment addition postulate: F is between C and A
7. $AD + DB = CF + FA$	substitution: lines 2, 5 and 6
8. $DB = FA$	subtraction property of equality: lines 4 and 7
9. $\overline{AF} \cong \overline{BD}$	definition of congruent segments: line 8
10. $\overline{AD} \cong \overline{BE}$	given
11. $\angle DAF \cong \angle EBD$	given: $\angle DAF$ is $\angle A$ and $\angle EBD$ is $\angle B$
12. $\triangle ADF \cong \triangle BED$	SAS postulate: lines 10, 11 and 9

Line 11 renames the given angles: D and F lie on the sides of $\angle A$, and E and D on the sides of $\angle B$. $\angle DAF$ is included between $\overline{AD}$ and $\overline{AF}$, and $\angle EBD$ between $\overline{BE}$ and $\overline{BD}$, so this is SAS. The pairing is $A \leftrightarrow B$, $D \leftrightarrow E$, $F \leftrightarrow D$.

SCORING

Not graded for points (homework). A complete proof gets $\overline{AF} \cong \overline{BD}$ from the segment addition postulate and the subtraction property of equality, takes $\overline{AD} \cong \overline{BE}$ and the angles at A and B from the given, and cites SAS with each angle between its two sides and the vertices in matching order.

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