AA similarity

If two angles of one triangle match two angles of another, the triangles are similar. No side has to be measured.

WORDS TO KNOW

Similar triangles: matching angles are equal, and matching sides are all in one ratio.

Similarity statement: $\triangle ABC \sim \triangle DEF$. The order of the letters gives the matching: A and D first, B and E second, C and F third.

Arc marks: one arc goes with one arc, two arcs with two arcs. They show which angles match.

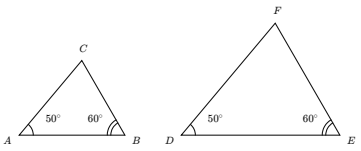
Scale factor k: the ratio of a side of the second triangle to the matching side of the first.

Congruent angles ($\cong$): angles with the same measure.

KEY IDEAS

1. Two angles settle the shape

Two angles and the side between them fix a triangle. A longer side gives a bigger copy of the same shape. The third angle comes free, since the angles add to 180°.



AA similarity. If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

2. Once they are similar, the sides follow

Match sides in the order of the similarity statement. One matching pair with both lengths known gives k, and k gives every other side.

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

3. Where pairs of equal angles come from

Where the pair comes from	Why the angles are equal
a corner the two triangles share	it is the same angle
two lines crossing	vertical angles
a line parallel to a side	corresponding or alternate interior angles
a right angle in each	every right angle is 90°
two angles already known	the third is 180° minus the other two

WATCH OUT

- **One pair of equal angles is not enough.** Find each third angle first. It may give a second pair, or show there is none.
- **Match by the arcs and the letters, not by the picture.** A triangle may be drawn turned or flipped.
- **Use the whole side.** When a small triangle sits inside a big one, the big side is the whole segment, such as $AB = AD + DB$.
- **AA is for triangles only.** A square and a long rectangle have all angles equal but are not similar.

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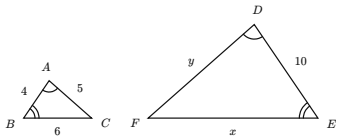
WORKED EXAMPLES

Example 1. In $\triangle ABC$, $\angle A = 46^\circ$ and $\angle B = 81^\circ$. In $\triangle PQR$, $\angle P = 53^\circ$ and $\angle Q = 46^\circ$. Are they similar?

1. Find the third angles: $\angle C = 180^\circ - 46^\circ - 81^\circ = 53^\circ$ and $\angle R = 180^\circ - 53^\circ - 46^\circ = 81^\circ$.
2. Now $\angle A = \angle Q$ and $\angle C = \angle P$. Two pairs, so the triangles are similar by AA.
3. Write the matching in order: A with Q, C with P, so B goes with R.

Answer: yes, $\triangle ABC \sim \triangle QRP$. **Check:** $\angle B = 81^\circ = \angle R$, so the third pair agrees.

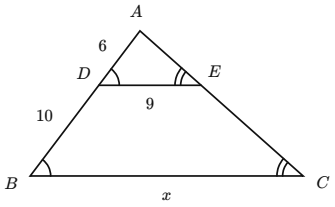
Example 2. $\angle A \cong \angle D$ and $\angle B \cong \angle E$. $AB = 4$, $BC = 6$, $CA = 5$ and $DE = 10$. Find EF and FD.



1. Two pairs of equal angles, so $\triangle ABC \sim \triangle DEF$ by AA.
2. AB and DE match and both are known: $k = \dfrac{10}{4} = \dfrac{5}{2}$.
3. EF matches BC: $EF = \dfrac{5}{2} \cdot 6 = 15$. FD matches CA: $FD = \dfrac{5}{2} \cdot 5 = 12.5$.

Answer: $EF = 15$, $FD = 12.5$. **Check:** $\dfrac{15}{6} = \dfrac{12.5}{5} = 2.5 = \dfrac{10}{4}$.

Example 3. In $\triangle ABC$, D is on AB and E is on AC with $DE \parallel BC$. $AD = 6$, $DB = 10$ and $DE = 9$. Find BC.



1. The angle at A is in both $\triangle ADE$ and $\triangle ABC$.
2. $DE \parallel BC$ gives $\angle ADE \cong \angle ABC$ (corresponding angles).
3. Two pairs, so $\triangle ADE \sim \triangle ABC$ by AA.
4. The side matching AD is the whole of $AB = 6 + 10 = 16$, so $k = \dfrac{16}{6} = \dfrac{8}{3}$.
5. BC matches DE: $BC = \dfrac{8}{3} \cdot 9 = 24$.

Answer: $BC = 24$. **Check:** $\dfrac{24}{9} = \dfrac{8}{3} = \dfrac{16}{6}$.

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