Geometry Unit 7 worksheet: AA similarity (answer key)

ANSWERS AT A GLANCE

1. $\{1, 2, 7\}$, $\{3, 4\}$, $\{5, 6\}$ and $\{8\}$.
2. $\angle S = 51°$, with $\triangle ABC \sim \triangle RST$; or $\angle S = 95°$, with $\triangle ABC \sim \triangle RTS$.
3. (a) $\triangle AED \sim \triangle ABC$, from $\angle AED \cong \angle ABC$ and the shared $\angle A$. (b) $AC = 20$, $EC = 14$. (c) $DE = 7.2$
4. (a) $\angle C = 60°$ and $\angle D = 72°$. The statement is wrong; correct is $\triangle ABC \sim \triangle EDF$. (b) $DF = 12$ and $EF = 15.4$.
5. (a) Yes: both have angles 100°, 40°, 40°. (b) No: one could have 40°, 70°, 70° and the other 40°, 40°, 100°.
6. 66 cm
7. $s = 4$

WORKED SOLUTIONS

1. Each of the eight triangles below is given by two of its angles.

Triangle 1: 40° and 75°. Triangle 2: 65° and 75°. Triangle 3: 40° and 70°. Triangle 4: 70° and 70°.

Triangle 5: 90° and 35°. Triangle 6: 55° and 35°. Triangle 7: 40° and 65°. Triangle 8: 75° and 35°.

Sort the triangles into groups so that any two triangles in the same group are similar and any two in different groups are not. A group may have only one triangle.

ANSWER

$\{1, 2, 7\}$, $\{3, 4\}$, $\{5, 6\}$ and $\{8\}$.

SOLUTION

Find the third angle of each triangle with the angle sum, 180° minus the two given angles:

Triangle 1: 65°; Triangle 2: 40°; Triangle 3: 70°; Triangle 4: 40°; Triangle 5: 55°; Triangle 6: 90°; Triangle 7: 75°; Triangle 8: 70°.

Now list each triangle's three angles:

- Triangles 1, 2 and 7 all have 40°, 65° and 75°.
- Triangles 3 and 4 both have 40°, 70° and 70°.
- Triangles 5 and 6 both have 35°, 55° and 90°.
- Triangle 8 has 35°, 70° and 75°, a list no other triangle has.

Triangles with the same three angles have two (in fact three) pairs of congruent angles, so they are similar by AA. Triangles with different lists cannot be matched angle for angle, so they are not similar. Triangle 8 shares one angle with several others but never two, so it stands alone.

2. In $\triangle ABC$, $\angle A = 34°$ and $\angle B = 51°$. In $\triangle RST$, $\angle R = 34°$. Find every measure $\angle S$ could have that makes the two triangles similar. For each one, write a similarity statement.

ANSWER

$\angle S = 51°$, with $\triangle ABC \sim \triangle RST$; or $\angle S = 95°$, with $\triangle ABC \sim \triangle RTS$.

SOLUTION

The angle sum gives $\angle C = 180° - 34° - 51° = 95°$, so the angles of $\triangle ABC$ are 34°, 51° and 95°.

$\angle A \cong \angle R$ is one pair. AA needs one more, so $\angle S$ must match one of the other two angles of $\triangle ABC$.

If $\angle S = 51°$, then $\angle S \cong \angle B$, and by AA $\triangle ABC \sim \triangle RST$ (T then measures 95°, matching C).

If $\angle S = 95°$, then $\angle S \cong \angle C$, and by AA $\triangle ABC \sim \triangle RTS$ (T then measures 51°, matching B).

No other value works: $\angle S = 34°$ would give $\triangle RST$ two 34° angles, a list $\triangle ABC$ does not have, and any other value matches nothing in $\triangle ABC$.

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3. In $\triangle ABC$, point D lies on $\overline{AB}$ and point E lies on $\overline{AC}$, and $\angle AED \cong \angle ABC$. (The segment $\overline{DE}$ is not parallel to $\overline{BC}$.) $AE = 6$, $AD = 8$, $AB = 15$ and $BC = 18$.

(a) Write a similarity statement for $\triangle AED$ and $\triangle ABC$, with the vertices in matching order, and give the two pairs of angles that make it true.

(b) Find AC and EC.

(c) Find DE.

ANSWER

(a) $\triangle AED \sim \triangle ABC$, from $\angle AED \cong \angle ABC$ and the shared $\angle A$. (b) $AC = 20$, $EC = 14$. (c) $DE = 7.2$

SOLUTION

(a) The two triangles share $\angle A$, and $\angle AED \cong \angle ABC$ is given. Two pairs of congruent angles, so by AA

$$\triangle AED \sim \triangle ABC$$

with A matching A, E matching B and D matching C. The match is not D with B: the congruent angle of the small triangle sits at E, on side $\overline{AC}$.

(b) Reading the letters, AE matches AB, AD matches AC, and ED matches BC. The pair with both lengths known gives the scale factor from $\triangle ABC$ to $\triangle AED$:

$$k = \frac{AE}{AB} = \frac{6}{15} = \frac{2}{5}$$

AD matches AC, so $8 = \frac{2}{5} \cdot AC$ and $AC = 20$. Then $EC = AC - AE = 20 - 6 = 14$.

(c) DE matches BC, so $DE = \frac{2}{5} \cdot 18 = 7.2$.

Check: $\frac{6}{15} = \frac{8}{20} = \frac{7.2}{18} = 0.4$.

4. In $\triangle ABC$, $\angle A = 48^\circ$ and $\angle B = 72^\circ$. In $\triangle DEF$, $\angle E = 48^\circ$ and $\angle F = 60^\circ$. A student writes $\triangle ABC \sim \triangle DEF$.

(a) Find the missing angles. Is the student's statement right? If not, write a correct similarity statement.

(b) $AB = 7$, $BC = 6$, $CA = 7.7$ and $DE = 14$. Find DF and EF.

ANSWER

(a) $\angle C = 60^\circ$ and $\angle D = 72^\circ$. The statement is wrong; correct is $\triangle ABC \sim \triangle EDF$. (b) $DF = 12$ and $EF = 15.4$.

SOLUTION

(a) The angle sum gives $\angle C = 180^\circ - 48^\circ - 72^\circ = 60^\circ$ and $\angle D = 180^\circ - 48^\circ - 60^\circ = 72^\circ$.

The triangles are similar by AA, but the matching goes by the angles: $\angle A$ and $\angle E$ are both 48°, $\angle B$ and $\angle D$ are both 72°, and $\angle C$ and $\angle F$ are both 60°. The statement $\triangle ABC \sim \triangle DEF$ would pair A (48°) with D (72°), so it is wrong. The correct statement is

$$\triangle ABC \sim \triangle EDF$$

(b) From the statement, AB matches ED, BC matches DF, and CA matches FE. The pair with both lengths known is AB and ED:

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$$k = \frac{ED}{AB} = \frac{14}{7} = 2$$

So $DF = 2 \cdot 6 = 12$ and $EF = 2 \cdot 7.7 = 15.4$.

With the student's statement one would pair BC with EF and get $EF = 12$, which is why the matching must be fixed first.

5. (a) Two isosceles triangles each have a 100° angle. Must they be similar? Explain.

(b) Two isosceles triangles each have a 40° angle. Must they be similar? Explain.

ANSWER

(a) Yes: both have angles 100°, 40°, 40°. (b) No: one could have 40°, 70°, 70° and the other 40°, 40°, 100°.

SOLUTION

(a) An isosceles triangle has two equal base angles. The 100° angle cannot be a base angle: two of them would already add to 200°, more than 180°. So 100° is the angle between the equal sides, and the base angles share the other 80° equally, 40° each. Both triangles have angles 100°, 40° and 40°, so two pairs of angles are congruent and they are similar by AA.

(b) A 40° angle can sit in either place. If it is the angle between the equal sides, the base angles are $\frac{180 - 40}{2} = 70^\circ$ each, giving 40°, 70°, 70°. If it is a base angle, the other base angle is also 40° and the third is 180° − 80° = 100°, giving 40°, 40°, 100°. These two triangles cannot be matched angle for angle, so they are not similar. The triangles need not be similar.

6. Lily has a folding camp stool. Its two straight legs cross at a hinge. The top ends of the legs hold the two edges of the seat, which is level, 33 cm apart. Each leg measures 30 cm from its top end to the hinge and 60 cm from the hinge to its foot on the level floor. How far apart are the two feet?

ANSWER

66 cm

SOLUTION

Call the top ends P and Q and the feet S and R, so that one leg is $\overline{PS}$, the other is $\overline{QR}$, and they cross at the hinge H. The seat $\overline{PQ}$ and the floor $\overline{RS}$ are both level, so they are parallel.

$\angle PHQ$ and $\angle SHR$ are vertical angles, so they are congruent. Line PS crosses the parallel lines PQ and RS, so $\angle HPQ \cong \angle HSR$ as alternate interior angles. Two pairs of congruent angles, so by AA

$$\triangle HPQ \sim \triangle HSR$$

with P matching S and Q matching R.

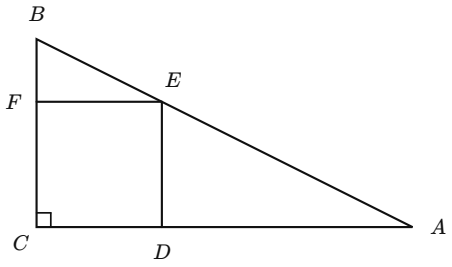
$HP = 30$ cm matches $HS = 60$ cm, so

$$k = \frac{60}{30}$$

and RS, which matches PQ, is $\frac{60}{30} \cdot 33 = 66$ cm.

7. In right triangle ABC, $\angle C = 90^\circ$, $AC = 12$ and $BC = 6$. A square $CDEF$ fits in the corner at C: D lies on $\overline{CA}$, F lies on $\overline{CB}$, and E lies on $\overline{AB}$. Find the side length of the square. (Call it s, and look for a triangle similar to $\triangle ACB$.)

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ANSWER

$s = 4$

SOLUTION

Compare $\triangle ADE$ with $\triangle ACB$. The angle at A belongs to both. $\angle ADE = 90^\circ$, because DE is a side of the square and D is on CA, and $\angle ACB = 90^\circ$. Two pairs of congruent angles, so by AA

$$\triangle ADE \sim \triangle ACB$$

with A matching A, D matching C and E matching B.

The sides of the small triangle are $DE = s$ and $AD = AC - DC = 12 - s$. AD matches AC and DE matches CB, so

$$\frac{12 - s}{12} = \frac{s}{6}$$

Multiply both sides by 12: $12 - s = 2s$, so $3s = 12$ and $s = 4$.

Check: $AD = 8$ and $DE = 4$, and $\frac{8}{12} = \frac{4}{6}$.

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