Finding a missing side with a trig ratio

You know one acute angle of a right triangle and one side. How do you find the other sides?

WORDS TO KNOW

Opposite, adjacent, hypotenuse: the three sides, named from the acute angle you know.

Trig ratio: $\sin A = \dfrac{\text{opp}}{\text{hyp}}$, $\cos A = \dfrac{\text{adj}}{\text{hyp}}$, $\tan A = \dfrac{\text{opp}}{\text{adj}}$.

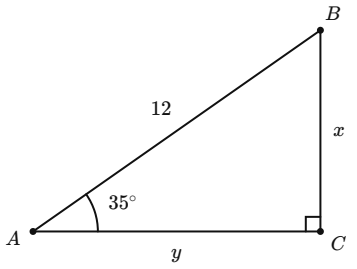
Solve a triangle: find every side and angle that is not given.

Degree mode: the calculator setting for angles in degrees. Every angle here is in degrees.

KEY IDEAS

1. Pick the one ratio that holds both sides

Stand at the angle you know and name the sides from there. Each ratio uses two of the three sides, and each pair of sides is in exactly one ratio.



Known and wanted sides	Ratio to use
opposite and hypotenuse	$\sin A$
adjacent and hypotenuse	$\cos A$
opposite and adjacent	$\tan A$

2. Write the equation and solve it

$\sin 35^\circ$ is just a number (about 0.5736), so the equation is a plain linear equation.

Unknown on top: multiply. $\sin 35^\circ = \dfrac{x}{12}$, so $x = 12 \sin 35^\circ \approx 6.88$.

Unknown underneath: divide. $\sin 35^\circ = \dfrac{6}{h}$, so $h = \dfrac{6}{\sin 35^\circ} \approx 10.46$.

The other angle: the two acute angles add to 90°. No trig needed.

3. Round once, at the end

Use the calculator's full value and round only the final answer. Where you can, find each side from the **given** side, not from a side you already rounded.

WATCH OUT

- **Name sides from the known angle.** A side "next to" the angle may be the hypotenuse, which is never called adjacent.
- **Divide, do not multiply, when the unknown is underneath.** If a hypotenuse comes out shorter than a leg, you had it upside down.
- **Degree mode.** In radian mode, $\sin 35 \approx -0.428$. A negative ratio of two lengths is the giveaway.
- **You need a side, and it must be named.** Angles alone fix the shape, not the size. "A side of 9" could be either leg or the hypotenuse.

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WORKED EXAMPLES

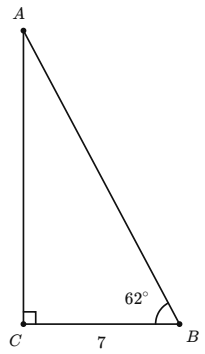
Example 1. In a right triangle, an acute angle is $28°$ and the leg opposite it is 9. Find the hypotenuse and the other leg.

1. Opposite and hypotenuse: use the sine. $\sin 28° = \dfrac{9}{h}$, unknown underneath, so $h = \dfrac{9}{\sin 28°} \approx 19.17$.
2. The other leg a is adjacent. Opposite and adjacent: use the tangent, from the given 9.

$$\tan 28° = \frac{9}{a} \qquad a = \frac{9}{\tan 28°} \approx 16.93$$

Answer: hypotenuse ≈ 19.17, other leg ≈ 16.93. **Check:** $9^2 + 16.93^2 \approx 367.6$ and $19.17^2 \approx 367.5$, equal up to rounding.

Example 2. In $\triangle ABC$ the right angle is at C, $\angle B = 62°$ and $BC = 7$. Solve the triangle.



1. The angle: $\angle A = 90° - 62° = 28°$.
2. Name sides from B: BC is adjacent, AC is opposite, AB is the hypotenuse.
3. AC (opposite) from BC (adjacent): $\tan 62° = \dfrac{AC}{7}$, so $AC = 7 \tan 62° \approx 13.17$.
4. AB (hypotenuse) from BC (adjacent): use the cosine.

$$\cos 62° = \frac{7}{AB} \qquad AB = \frac{7}{\cos 62°} \approx 14.91$$

Answer: $\angle A = 28°$, $AC \approx 13.17$, $AB \approx 14.91$. **Check:** $7^2 + 13.17^2 \approx 222.4$ and $14.91^2 \approx 222.3$. The longer leg is across from the larger angle.

Example 3. A ladder leans against a wall on level ground. Its foot is 1.5 m from the wall and it makes a $72°$ angle with the ground. How long is the ladder, and how high does it reach?

1. Wall, ground and ladder make a right triangle. The ladder is the hypotenuse. From the $72°$ angle, the 1.5 m is adjacent and the height is opposite.
2. Length L: adjacent and hypotenuse, so cosine. $L = \dfrac{1.5}{\cos 72°} \approx 4.85$ m.
3. Height h: opposite and adjacent, so tangent. $h = 1.5 \tan 72° \approx 4.62$ m.

Answer: the ladder is about 4.85 m long and reaches about 4.62 m up. **Check:** the height is less than the ladder, as a leg must be.

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