Geometry Unit 8 worksheet: Solving right triangles (answer key)

ANSWERS AT A GLANCE

1. (a) $x \approx 16.57$ (b) $x \approx 8.12$ (c) $x \approx 20.52$ (d) $x \approx 19.27$ (e) $x \approx 9.92$ (f) $x \approx 0.64$
2. The tip is 10.7 m above the hinge and 11.9 m out from it horizontally.
3. The perimeter is about 25.2.
4. Height ≈ 6.29, $CD \approx 13.86$, area ≈ 106.52.
5. (a) $AB = \dfrac{b}{\cos\theta}$, $BC = b\tan\theta$. (b) $AB = \dfrac{b}{\sin(90^\circ - \theta)}$. (c) $\sin(90^\circ - \theta) = \cos\theta$, because the side opposite $\angle B$ is the side adjacent to $\angle A$.
6. $AC \approx 10.73$, $BC \approx 7.24$, $AB \approx 12.94$.
7. (a) $\overline{QR}$ does not touch P; it is opposite $\angle P$. (b) $QR \approx 5.07$. (c) $\overline{PR}$.
8. (a) About 5.66 m. (b) About 3.00 m.

WORKED SOLUTIONS

1. Solve each equation for x. Give x to two decimal places.

(a) $\cos 23^\circ = \dfrac{x}{18}$ (b) $\tan 61^\circ = \dfrac{x}{4.5}$ (c) $\sin 17^\circ = \dfrac{6}{x}$

(d) $\tan 34^\circ = \dfrac{13}{x}$ (e) $\cos 76^\circ = \dfrac{2.4}{x}$ (f) $\sin 49^\circ = \dfrac{x}{0.85}$

ANSWER

(a) $x \approx 16.57$ (b) $x \approx 8.12$ (c) $x \approx 20.52$ (d) $x \approx 19.27$ (e) $x \approx 9.92$ (f) $x \approx 0.64$

SOLUTION

The ratio on the left is a fixed number, so each is a linear equation in x.

(a) The unknown is on top: multiply by 18. $x = 18\cos 23^\circ \approx 16.57$.

(b) Multiply by 4.5: $x = 4.5\tan 61^\circ \approx 8.12$.

(c) The unknown is underneath: multiply by x, then divide by $\sin 17^\circ$. $x = \dfrac{6}{\sin 17^\circ} \approx 20.52$.

(d) $x = \dfrac{13}{\tan 34^\circ} \approx 19.27$.

(e) $x = \dfrac{2.4}{\cos 76^\circ} \approx 9.92$.

(f) Multiply by 0.85: $x = 0.85\sin 49^\circ \approx 0.64$.

2. The boom of a crane is a straight arm 16 m long, hinged at its lower end. It is raised to an angle of $42°$ above the horizontal. How far above the hinge is the tip of the boom, and how far from the hinge is it, measured horizontally? Give both to the nearest tenth of a meter.

ANSWER

The tip is 10.7 m above the hinge and 11.9 m out from it horizontally.

SOLUTION

The boom, the horizontal line through the hinge and the vertical line down from the tip make a right triangle, with the right angle on the horizontal line directly below the tip. The boom is the hypotenuse. From the $42°$ angle at the hinge, the height of the tip is opposite and the horizontal reach is adjacent.

Height: opposite and hypotenuse, so the sine. $\sin 42^\circ = \dfrac{h}{16}$, so $h = 16\sin 42^\circ \approx 10.7$ m.

Reach: adjacent and hypotenuse, so the cosine. $\cos 42^\circ = \dfrac{d}{16}$, so $d = 16\cos 42^\circ \approx 11.9$ m.

Both legs come from the given 16 m, and both are shorter than the boom, as they must be.

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3. In $\triangle JKL$ the right angle is at K, $m\angle L = 58^\circ$ and $JK = 9$. Find the perimeter of $\triangle JKL$ to the nearest tenth.

ANSWER

The perimeter is about 25.2.

SOLUTION

Name the sides from $\angle L$, the known angle. $\overline{JK}$ does not touch L, so it is opposite $\angle L$. $\overline{KL}$ runs from L to the right angle, so it is adjacent. $\overline{JL}$ is the hypotenuse.

KL: opposite and adjacent, so the tangent, with the unknown underneath. $\tan 58^\circ = \dfrac{9}{KL}$, so $KL = \dfrac{9}{\tan 58^\circ}$.

JL: opposite and hypotenuse, so the sine, again underneath. $\sin 58^\circ = \dfrac{9}{JL}$, so $JL = \dfrac{9}{\sin 58^\circ}$.

Add the three sides, keeping the calculator's full values and rounding once:

$$9 + \frac{9}{\tan 58^\circ} + \frac{9}{\sin 58^\circ} \approx 25.2$$

4. Isosceles trapezoid $ABCD$ has the longer base $AB = 20$, legs $AD = BC = 7$, and base angles $m\angle A = m\angle B = 64^\circ$. Find its height, the shorter base CD, and its area, each to two decimal places.

ANSWER

Height ≈ 6.29, $CD \approx 13.86$, area ≈ 106.52.

SOLUTION

Drop perpendiculars from D and C to $\overline{AB}$, meeting it at E and F. Right triangles AED and BFC each have hypotenuse 7 and an acute angle of 64° at A or B, and $DCFE$ is a rectangle, so $EF = CD$.

Height: DE is opposite the 64° angle, so $DE = 7\sin 64^\circ \approx 6.29$.

The overhang AE is adjacent to the 64° angle, so $AE = 7\cos 64^\circ \approx 3.0686$, and BF is the same. Then

$$CD = 20 - 2 \cdot 7\cos 64^\circ \approx 13.86$$

Area, from the unrounded values:

$$\frac{1}{2}\left(20 + 20 - 14\cos 64^\circ\right) \cdot 7\sin 64^\circ \approx 106.52$$

5. In right $\triangle ABC$ the right angle is at C, $m\angle A = \theta$, and $AC = b$.

(a) Name the sides from $\angle A$, and write AB and BC in terms of b and θ.

(b) Now name the sides from $\angle B$, whose measure is $90^\circ - \theta$, and write AB again in terms of b and θ.

(c) Explain why your two expressions for AB are equal.

ANSWER

(a) $AB = \dfrac{b}{\cos\theta}$, $BC = b\tan\theta$. (b) $AB = \dfrac{b}{\sin(90^\circ - \theta)}$. (c) $\sin(90^\circ - \theta) = \cos\theta$, because the side opposite $\angle B$ is the side adjacent to $\angle A$.

SOLUTION

(a) From A: $\overline{AC}$ runs from A to the right angle, so it is adjacent; $\overline{BC}$ is opposite; $\overline{AB}$ is the hypotenuse.

Adjacent and hypotenuse means the cosine, with the unknown underneath: $\cos\theta = \dfrac{b}{AB}$, so $AB = \dfrac{b}{\cos\theta}$.

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Opposite and adjacent means the tangent: $\tan\theta = \dfrac{BC}{b}$, so $BC = b\tan\theta$.

(b) From B: $\overline{AC}$ does not touch B, so it is now opposite. Opposite and hypotenuse means the sine: $\sin(90^\circ - \theta) = \dfrac{b}{AB}$, so $AB = \dfrac{b}{\sin(90^\circ - \theta)}$.

(c) Both expressions describe the same segment, so they must agree. They do because $\sin(90^\circ - \theta) = \dfrac{AC}{AB} = \cos\theta$: the side opposite one acute angle is the side adjacent to the other, so the sine of one acute angle is the cosine of its complement.

6. In right $\triangle ABC$ the right angle is at C, $m\angle A = 34^\circ$, and the altitude $\overline{CD}$ to the hypotenuse has length 6. Find AC, BC and AB to two decimal places.

ANSWER

$AC \approx 10.73$, $BC \approx 7.24$, $AB \approx 12.94$.

SOLUTION

The altitude makes two right triangles, ACD and BCD, both with the right angle at D, sharing the leg $CD = 6$. In $\triangle ABC$, $m\angle B = 90^\circ - 34^\circ = 56^\circ$.

In $\triangle ACD$: CD is opposite $\angle A$ and AC is the hypotenuse, so $AC = \dfrac{6}{\sin 34^\circ} \approx 10.73$. AD is adjacent to $\angle A$, so $AD = \dfrac{6}{\tan 34^\circ} \approx 8.8953$.

In $\triangle BCD$: CD is opposite $\angle B$, so $BC = \dfrac{6}{\sin 56^\circ} \approx 7.24$ and $DB = \dfrac{6}{\tan 56^\circ} \approx 4.0471$.

$AB = AD + DB = \dfrac{6}{\tan 34^\circ} + \dfrac{6}{\tan 56^\circ} \approx 12.94$.

Check: $10.73^2 + 7.24^2 \approx 167.5 \approx 12.94^2$, and $AD \cdot DB \approx 36.0 = CD^2$, the altitude rule from earlier in the unit.

7. In $\triangle PQR$ the right angle is at R, $m\angle P = 25^\circ$ and $PQ = 12$. Mia wants QR. She writes "QR is next to the angle, so it is adjacent" and gets $QR = 12\cos 25^\circ \approx 10.88$.

(a) What is wrong with Mia's naming?

(b) Find QR correctly, to two decimal places.

(c) Mia's number is the length of a side of the triangle. Which side?

ANSWER

(a) $\overline{QR}$ does not touch P; it is opposite $\angle P$. (b) $QR \approx 5.07$. (c) $\overline{PR}$.

SOLUTION

(a) Name the sides from P. The adjacent side runs from P to the right angle at R; that is $\overline{PR}$. $\overline{QR}$ lies across the triangle from P, so it is opposite $\angle P$. $\overline{QR}$ is next to the right angle, which is not the angle the names come from.

(b) Opposite and hypotenuse means the sine: $\sin 25^\circ = \dfrac{QR}{12}$, so $QR = 12\sin 25^\circ \approx 5.07$.

(c) $12\cos 25^\circ$ is the side adjacent to $\angle P$, which is $PR \approx 10.88$. A sense check also flags her answer: $\angle P = 25^\circ$ is the smaller acute angle, so the side opposite it should be the shorter leg, and 10.88 is longer than 5.07.

8. The end of a house's attic is an isosceles triangle. Its base is the attic floor, 9.6 m wide, and each of the two roof slopes meets the floor at an angle of 32°. The rafters run straight from the ends of the floor up to the ridge.

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(a) How long is each rafter?

(b) How high is the ridge above the attic floor?

Give both to two decimal places.

ANSWER

(a) About 5.66 m. (b) About 3.00 m.

SOLUTION

Drop the perpendicular from the ridge to the floor. It cuts the isosceles triangle into two right triangles with equal hypotenuses (the rafters) and a shared leg, so they are congruent (hypotenuse and leg), and the foot of the perpendicular is the midpoint of the floor. Each right triangle has a horizontal leg of $9.6 \div 2 = 4.8$ m, adjacent to the 32° angle.

(a) The rafter is the hypotenuse; adjacent and hypotenuse means the cosine, with the unknown underneath:

$$\cos 32^\circ = \frac{4.8}{r} \quad \Longrightarrow \quad r = \frac{4.8}{\cos 32^\circ} \approx 5.66 \text{ m}$$

(b) The height is opposite the 32° angle; opposite and adjacent means the tangent, from the given 4.8:

$$\tan 32^\circ = \frac{h}{4.8} \quad \Longrightarrow \quad h = 4.8 \tan 32^\circ \approx 3.00 \text{ m}$$

Check: $4.8^2 + 2.99937\ldots^2 \approx 32.04$ and $5.66006\ldots^2 \approx 32.04$.

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