Multiplying with partial products

A section has 6 rows of 34 seats. That is not a fact you know by heart. How do you break a big multiplication into small ones you do know?

WORDS TO KNOW

Factors: the numbers you multiply. **Product:** the answer.

Area model: a rectangle whose sides are the factors. Its area is the product.

Lattice: a grid with diagonal lines that records the same partial products.

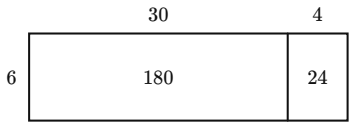
Partial product: the answer to one small piece of a bigger multiplication.

Distributive property: multiplying a sum is the same as multiplying each part and adding. For example, $6 \times (30 + 4)$ is $6 \times 30 + 6 \times 4$.

KEY IDEAS

1. Split a factor by place value, multiply each part, add

Split 34 into 30 + 4. Cut the rectangle there. Each piece is a basic fact, maybe shifted.



$$6 \times 34 = 6 \times 30 + 6 \times 4 = 180 + 24 = 204$$

The pieces are added, never multiplied.

2. Longer numbers: one piece for each place

Write the long factor in expanded form and multiply each part. In a column, line every partial product up by place value, then add.

$6 \times 3{,}214 = 18{,}000 + 1{,}200 + 60 + 24 = 19{,}284$

A zero digit gives a piece of 0. You may skip it, as long as every other piece is written in full.

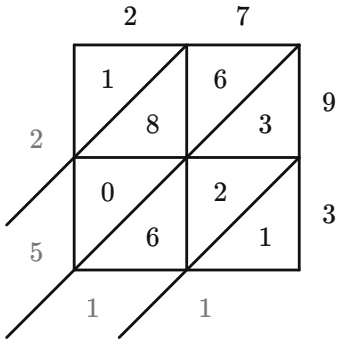
3 2 1 4	
× 6	
2 4	6×4
6 0	6×10
1 2 0 0	6×200
1 8 0 0 0	$6 \times 3{,}000$
1 9 2 8 4	

3. Two digits by two digits: four pieces

Split both factors. Each part of one factor meets each part of the other, so 2 parts times 2 parts gives 4 pieces. Adding a row of the area model gives one factor times one part of the other.

4. A lattice keeps the places on diagonals

In each cell, write the product of the two digits that meet there, tens above the diagonal and ones below. A product under 10 gets a 0 on top, like 06. Add along each diagonal band from the bottom right. If a band makes 10 or more, write its ones digit and pass its tens digit on to the next band.



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WATCH OUT

- **Every part meets every part.** In a two-digit by two-digit product, leaving out a piece leaves part of the rectangle uncounted. There are four pieces, not two.
- **Keep each piece in its place.** 2 × 700 is 1,400, not 140. Write each piece in full and line it up.
- **Check with an estimate.** Round each factor and multiply. An answer a whole place too big or too small will not fit.

WORKED EXAMPLES

Example 1. Find 7 times 86 with partial products.

1. Split 86 by place value: 86 = 80 + 6.
2. Multiply each part by 7: 7 × 80 = 560 (that is 56 tens) and 7 × 6 = 42.
3. Add: 560 + 42. The tens make 6 + 4 = 10 tens, which is one more hundred. So the sum is 602.

Answer: 7 × 86 = 602. **Check:** 7 × 90 = 630, and 86 is a bit less than 90, so the answer should be a bit less than 630. It is.

Example 2. A farm has 7 fields with 3,401 plants in each field. How many plants is that?

1. Expanded form: 3,401 = 3,000 + 400 + 0 + 1. There are no tens.
2. Multiply each part by 7: 7 × 3,000 = 21,000, 7 × 400 = 2,800, the tens piece is 0, and 7 × 1 = 7.
3. Add the pieces that are not zero: 21,000 + 2,800 + 7 = 23,807.

Answer: 23,807 plants. **Check:** the biggest piece is 21,000, and the product is a bit more.

Example 3. Find 87 times 25 with a four-part area model.

×	80	7
20	1,600	140
5	400	35

1. Split both factors: 87 = 80 + 7 and 25 = 20 + 5. Each cell of the model is its column number times its row number.
2. Add by rows. The row for 20: 1,600 + 140 = 1,740. The row for 5: 400 + 35 = 435.
3. Add the rows: 1,740 + 435 = 2,175.

Answer: 87 × 25 = 2,175. **Check:** add by columns instead. The column for 80 is 1,600 + 400 = 2,000 and the column for 7 is 140 + 35 = 175, and 2,000 + 175 = 2,175.

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